THE CONVERSE OF A THEOREM OF TCHAPLYGIN ON DIFFERENTIAL INEQUALITIES

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1. **Introduction.** Suppose that $y(x)$ is a solution of the linear differential equation

$$y'' - p_1 y' - p_2 y - q = 0, \qquad x \geqq x_0, \tag{1.1}$$

where $p_1(x)$, $p_2(x)$ and $q(x)$ are continuous when $x \geqq x_0$, and that

$$y(x_0) = y_0, \qquad y'(x_0) = y_0'. \tag{1.2}$$

Then if $v(x)$ satisfies the differential inequality

$$v'' - p_1 v' - p_2 v - q > 0, \qquad x \geqq x_0, \tag{1.3}$$

and the same boundary conditions as $y(x)$ at x_0, it is clear that the inequality

$$v(x) > y(x) \tag{1.4}$$

holds in some right-hand neighborhood of x_0. Tchaplygin[1] has proved that the inequality (1.4) holds when $x_0 < x \leqq x_1$ provided that there exists a solution $\lambda(x)$ of the Riccati equation

$$\lambda' + \lambda^2 + p_1 \lambda + (p_1' - p_2) = 0 \tag{1.5}$$

which is continuous when $x_0 < x < x_1$. Let $X(x_0)$ be the least upper bound of values x_1 for which the Riccati equation admits a continuous solution when $x_0 < x < x_1$. Then the inequality (1.4) holds when $x_0 < x \leqq X(x_0)$, and Petrov [2] has shown that if p_1 and p_2 are constants no better result is true. That is, if p_1 and p_2 are constants and $X(x_0)$ is finite, then there exists a function $v(x)$ satisfying (1.3) and (1.2) for which $v(x) = y(x)$ at a point arbitrarily close to but greater than $X(x_0)$. It is the purpose of this paper to show that this last result is true without the restriction that p_1 and p_2 are constants. We prefer to state our results in terms of and make our proofs depend

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[1] The author knows this result only by virtue of a reference to it contained in the paper of Petrov [2], and there it is not made clear whether or not Tchaplygin took the interval from x_0 to x_1 to be open, as we have written it, or closed or half-open. It follows, however, from the results obtained in §2 that this statement is true for the open interval and hence is a fortiori true for the closed and half-open intervals. Numbers in brackets refer to the bibliography at the end of the paper.