SUBSERIES OF SERIES WHICH ARE NOT ABSOLUTELY CONVERGENT

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1. **Introduction.** It is the object of this note to give two theorems on series, of real or complex terms, which fail to converge absolutely. The second is a corollary of the first. They say, roughly, that each such series becomes or remains divergent after "nearly all" of its terms, suitably selected, are discarded or replaced by zeros.

THEOREM 1. *If $a(1)+a(2)+a(3)+\cdots$ is a series of real or complex terms which fails to converge absolutely, then there is an increasing sequence*

$$1 \leqq n_1 < n_2 < n_3 < \cdots \tag{1}$$

of integers such that $n_{k+1}-n_k \to \infty$ as $n \to \infty$ and the series

$$a(n_1) + a(n_2) + a(n_3) + \cdots \tag{2}$$

is divergent.

THEOREM 2. *If $a(1)+a(2)+a(3)+\cdots$ is a series of real or complex terms which fails to converge absolutely, then there is a sequence $x_1, x_2, x_3, \cdots$ of which each element is either 0 or 1, such that*

$$\lim_{n\to\infty} (x_1 + x_2 + \cdots + x_n)/n = 0 \tag{3}$$

and the series

$$a(1)x_1 + a(2)x_2 + a(3)x_3 + \cdots \tag{4}$$

is divergent.

2. **Series of non-negative terms.** In this section, we obtain the conclusions of Theorems 1 and 2 for the case in which $a(1)+a(2)+\cdots$ is a series of real non-negative terms which fails to converge absolutely and accordingly $a(1)+a(2)+\cdots$ is a divergent series of real non-negative terms. Choose integers

$$1 = \alpha_2 < \beta_2 < \alpha_3 < \beta_3 < \alpha_4 < \beta_4 < \cdots \tag{5}$$

such that, for each $p=2, 3, 4, \cdots$,

$$\sum_{k=\alpha_p}^{\beta_p-1} a(k) > p, \tag{6}$$

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