within any time Δt be respectively $k_{i+1}\Delta t+o(\Delta t)$ and $g_i\Delta t+o(\Delta t)$. Let the probability of any other transition during Δt be $o(\Delta t)$. Let k_i's and g_i's be constant. Let $P_i(t)$ be the probability of the existence of the state i at the time t if the state 0 existed at the time $t=0$. The paper gives $\int_0^\infty t^m P_i(t)dt$ or $\int_0^\infty t^m P_i'(t)dt$ as algebraic expressions of the k_i's and g_i's. The result is obtained by using the complete homogeneous symmetric functions of the poles of the Laplace transform of $P_i(t)$ or $P_i'(t)$. (Received November 20, 1946.)

91. Gerhard Tintner: *The statistical estimation of the dimensionality of a given set of observations.*

There is a set of N observations of p variables $X_{it}=M_{it}+y_{it}$, where M_{it} is the systematic part and y_{it} the random element $(i=1, 2, \cdots, p; t=1, 2, \cdots, N)$. The y_{it} are normally and independently distributed with means zero. There are in the population R linear independent relationships of the form: $k_{s0}+\sum_i k_{si}M_{it}=0$ $(s=1, 2 \cdots R)$. A method based upon results of R. A. Fisher (Annals of Eugenics, 1938) and P. L. Hsu (ibid. 1941) assumes that we have a large sample estimate of the covariance matrix of the y_{it}: $\|V_{ij}\|$. The determinantal equation is $|a_{ij}-\lambda V_{ij}|=0$, where the a_{ij} are the sample covariances of the X_{it}. λ_1 is the smallest root of the equation, λ_2 the next smallest, and so on. $\Lambda_r=(N-1)\sum_1^r\lambda_i$. These sums of squares are distributed like χ^2 with $r(N-p-1+r)$ degrees of freedom and may be used to estimate the number R of linear relations between the M_{it}. By inserting the R smallest roots into the determinantal equation matrices are found for the computation of the coefficients k_{si} (G. Tintner, Ann. of Math. Statist., 1945). (Received October 15, 1946.)

92. Jacob Wolfowitz: *On the efficiency of unbiased sequential estimates.*

Let $f(x, \theta)$ be the distribution function (density function or probability function) of a chance variable X, which depends upon the parameter $\theta=\theta_1, \cdots, \theta_l$. Let n successive independent observations be made on X, where n is itself a chance variable, and the decision to terminate the drawing of observations depends upon the observations already obtained. Let $\theta_1^*(x_1, \cdots, x_n), \cdots, \theta_l^*(x_1, \cdots, x_n)$ be joint unbiased estimates of $\theta_1, \cdots, \theta_l$. Let $\|\lambda_{ij}\|$ be the nonsingular matrix of their covariances, and $\|\lambda^{ij}\|$ its inverse. Under certain regularity conditions it is proved that the concentration ellipsoid $\sum\lambda^{ij}(k_i-\theta_i)(k_j-\theta_j)=l+2$ always contains within itself the ellipsoid $\sum\mu_{ij}(k_i-\theta_i)(k_j-\theta_j)=l+2$, where $\mu_{ij}=EnE((\partial \log f/\partial\theta_i)(\partial \log f/\partial\theta_j))$. (Received October 21, 1946.)

Topology

93. R. F. Arens: *Pseudo-normed algebras.*

A pseudo-norm defined on a linear algebra A over, for example, the reals, is a real-valued function having the formal properties of a norm in a normed ring except that the pseudo-norm of some nonzero elements may vanish. Consequently, a pseudo-normed algebra is required by definition to have a complete system of pseudo-norms. The continuity of inversion is considered; and the singular elements related to the closed divisorless proper ideals. The space of the latter, and conditions for its compactness, are considered. As a by-product a characterization of rings of all continuous complex-valued functions on a topological space is obtained. Special attention is given to the question of completeness of quotient rings. (Received November 19, 1946.)