case $N^2=0$ the four sufficient conditions can be stated in the following manner. Suppose that A/N has t simple ideals. Then the Cartan invariants are the elements of a t by t matrix C with integer elements. Set $D=C-I$. Associate a graph G of $2t$ vertices $P_1, \cdots, P_t \cdot Q_1, \cdots, Q_t$ with D by joining P_i and Q_j if and only if $d_{ij} \neq 0$. The four sufficient conditions are then (1) some $d_{ij} > 1$; (2) G is not a tree; (3) some vertex of G is of order greater than 3; (4) some connected subgraph of G has more than one vertex of order 3. (Received November 21, 1946.)

23. Bernard Vinograde. *Radicals associated with equivalent semi-simple residue systems.*

This paper investigates rings wherein the radical is a homomorphic additive image of the semi-simple part and satisfies $f(xy)=xf(y)+f(x)y+f(x)f(y)$, where f is the homomorphism. $f(xy)$ affords a trioperational approach. This is an aspect of the distribution of residue systems in a semi-primary ring. (Received October 24, 1946.)

24. Daniel Zelinsky: *Nonassociative valuations.*

An ordered quasigroup G is a quasigroup, written additively, which is linearly ordered by a transitive, binary relation $>$, having the property that $x>y$ implies $x+z>y+z$ and $z+x>z+y$ for all x, y, z of G. A valuation, V, of a (nonassociative) ring R is a function on R to an ordered quasigroup with ∞ adjoined such that for all a, b of R, $V(a+b) \geqq \min [V(a), V(b)]$, $V(ab)=V(a)+V(b)$, $V(a)=\infty$ if and only if $a=0$. The principal theorem of this paper is the following: If R is an algebra of finite order over a field F, if R has a unity quantity and if $V(F)$ is an archimedean-ordered group, then $V(R)$ is an archimedean-ordered abelian group in which $V(F)$ has finite index. Examples of nonassociative ordered loops are obtained by simple loop extensions. The existence of a ring with arbitrary prescribed value loop and residue-class ring (without zero divisors) is proved. From these two facts follow examples showing that the hypothesis "$V(F)$ is archimedean-ordered" cannot be omitted in the theorem above. This is in strong contrast with the associative, noncommutative case. (See O. F. G. Schilling, *Noncommutative valuations*, Bull. Amer. Math. Soc. vol. 51 (1945) pp. 297–304.) (Received November 13, 1946.)

ANALYSIS

25. H. W. Becker: *Generalizations of the Epstein-Fourier series.*

These series are combinations of exponential and Fourier series (Leo F. Epstein, Journal of Mathematics and Physics vol. 18 (1939) p. 60, (19)). Where $K_0=1$, $x=r\cos\theta$, $y=r\sin\theta$, and "soc" means "sine or cosine," some generalizations are: (1) $\exp[X+r \operatorname{soc}(\ )\theta \cdot (KX+Z)] = [\operatorname{soc}(Zy+e^{Xx}\sin Xy)] \cdot [\exp(Zx+e^{Xx}\cos Xy)]$, the $(KX+Z)$ except for change of sign of X being the polynomials of Steffensen (*Some recent researches in the theory of statistics and actuarial science*, Cambridge Press, 1930, p. 24); (2) $\exp[e\{1+r\operatorname{soc}(\ )\theta\cdot K^{(2)}\}] = [\operatorname{soc}\{e^{e^x\cos y}\sin(e^x\sin y)\}] \cdot [\exp\{e^{e^x\cos y}\cos(e^x\sin y)\}]$, the $K^{(2)}$ being Bell numbers (Ann. of Math. vol. 39 (1938) p. 539). Under the substitutions $r\to rX$, $X\to X^{-1}$, $Z=0$, (1) becomes (3) $\exp[X^{-1}+r\operatorname{soc}(\ )\theta\cdot T] = [\operatorname{soc}(e^x\sin y)]\cdot[\exp(e^x\cos y)]$, where $T=XKX^{-1}$ is an umbral transform of Riordan and Kaplansky (*The problem of the rooks and its applications*). It is noteworthy that the right side of (3) is free of X, endowing the left side with a kind of invariance. (Received October 1, 1946.)