IDEAL THEORY ON OPEN RIEMANN SURFACES

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Introduction. The theorems of the classical ideal theory in fields of algebraic numbers hold in rings of analytic functions on compact Riemann surfaces. The surfaces admitted in our discussion are closely related to algebraic surfaces; we deal either with compact surfaces from which a finite number of points are omitted or, more generally, with surfaces determined by an algebroid function. The local aspects of the resulting ideal theory are the same as those found in the theory of algebraic functions. However, the ideal theory in the large is quite different. We now cannot count on the simplifications which are implied by the ascending chain condition for ideals. The recent theories of functions on topological spaces provide the necessary tools for a simple theory in the large. We shall show that the topologization of rings of entire functions by means of the topology of the underlying surface furnishes a fruitful method. Thus, the closed maximal ideals will correspond to the points of the given surfaces. Finally, to quote another result, all closed ideals are principal.

1. **Some basic definitions.** Suppose that Σ is the complex number sphere whose points at finite distance $p=p_a$ are in 1-1 correspondence with the elements a of the complex number field C. We assume that each point p of Σ, the point at infinity included, has conformal neighborhoods. In general let t_p be a local uniformizing variable at p, that is, a function which affords the mapping of a conformal neighborhood of p in the unit circle $|t_p|<1$. We associate with each point p a model Σ_p of Σ and consider functions $f(z)$ on Σ with values in $\{\Sigma_p\}$, in other words, $f(p)\in\Sigma_p$ for $p\in\Sigma$. As usual we say that a function $f(z)$ is meromorphic on Σ if there exists for each point $p\in\Sigma$ a convergent expansion

$$f(z) = \sum_{i=m}^{\infty} c_{p,i}t_p^i, \qquad c_{p,i}\in C, \tag{1}$$

where the first nonvanishing coefficient $c_{p,m}$ has a finite integral subscript. Then all meromorphic functions on Σ form a field $F(\Sigma)$ which contains C [2, 17].[1]

Next we associate to each function $f(z)$ the integer $m=V_pf(z)$ and

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[1] The numbers in brackets refer to the list of references.