NONLINEAR NETWORKS. I

R. J. DUFFIN

The object of this note is to show that a certain system of nonlinear differential equations has a unique asymptotic solution, that is, all solutions approach each other as the independent variable becomes infinite. The interest of these equations is that they describe the vibrations of electrical networks so we shall first discuss the physical origin of the equations.

A linear network is a collection of linear inductors, linear resistors and linear capacitors arbitrarily interconnected. Suppose that such a network has no undamped free vibration. Then a given impressed force may give rise to more than one response but as time goes on the transient vibrations die out and there is a unique relation between impressed force and response. This, of course, is well known. Our main theorem states that if in such a network the linear resistors are replaced by *quasi-linear* resistors then again, after sufficient time has elapsed, there is a unique relation between impressed force and response.

A quasi-linear resistor is a conductor whose differential resistance lies between positive limits. No other nonlinearity besides this type of nonlinear damping is considered. Quasi-linear resistors have extensive practical application.

For example, consider a linear network with one degree of freedom. An inductor of inductance L, a resistor of resistance R and a capacitor of capacitance S^{-1} are connected in series. The current $y(t)$ flowing in this circuit must satisfy the following differential equation

$$L\frac{dy}{dt} + Ry + S\int ydt = e.$$

Here $e(t)$ is the electromotive force impressed in the circuit and may be an arbitrary function of time.

The corresponding nonlinear equation to be studied is obtained by replacing the linear relation Ry by a function $V(y)$ which for all values of y is such that $\delta \leqq V'(y) \leqq \Delta$, where δ and Δ are positive constants.

In the general network with n degrees of freedom a set of n independent circuits (meshes) is chosen. Then any distribution of cur-

Presented to the Society, February 23, 1946; received by the editors January 31, 1946.