ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

262. P. T. Bateman: *On the representations of a number as the sum of three squares.*

In determining $r_s(n)$, the number of representations of a positive integer n as the sum of s squares, it is known that the singular series $\rho_s(n)$ constructed by Hardy (Trans. Amer. Math. Soc. vol. 21 (1920) pp. 255–284) gives exact results for $3 \leqq s \leqq 8$. For $5 \leqq s \leqq 8$ Hardy proved this by showing that the function $\Psi_s(\tau) = 1 + \sum_{n+1}^{\infty} \rho_s(n) e^{\pi i \tau n}$, $\vartheta(\tau) > 0$, has exactly the same behavior under the modular subgroup Γ_3 as the function $\vartheta_3(0 | \tau)^s = (\sum_{n=-\infty}^{\infty} e^{\pi i \tau n^2})^s = 1 + \sum_{n=1}^{\infty} r_s(n) e^{\pi i \tau n}$. For $s = 3$, 4 the double series of partial fractions for $\Psi_s(\tau)$ which Hardy used to establish the modular properties of $\Psi_s(\tau)$ is no longer absolutely convergent, even though the proof is correct formally. For $s = 4$ absolute convergence is easily restored by grouping terms, but for $s = 3$ this is not possible. In this paper the case $s = 3$ is successfully treated by supplementing the Hardy method with a limit process of the kind used by Hecke in defining his generalized Eisenstein series. There are some analytical intricacies in applying the limit process, but no formal difficulties. A particular result is that for n square free, $r_3(n) = C\pi^{-1} n^{1/2} \cdot \sum_{k=1}^{\infty} (-n/k) k^{-1}$, where $C = 0$ if $n \equiv 7 \pmod 8$, $C = 16$ if $n \equiv 3 \pmod 8$, and $C = 24$ if $n \equiv 1, 2, 5, 6 \pmod 8$. (Received July 13, 1946.)

263. Garrett Birkhoff and P. M. Whitman: *Representation theory for certain non-associative algebras.*

It is known that the ways of embedding a Lie algebra L in a linear associative enveloping algebra are all obtained from a "universal" enveloping algebra $A_u(L)$ with infinite basis, by setting a suitable ideal of L equal to zero. It is shown that a corresponding theorem holds for any Jordan algebra J, but that $A_u(J)$ has a finite basis if J does. Particular examples are worked out for Lie and Jordan algebras. If L or J is "solvable," then the finite basis theorem is valid in $A_u(L)$ and $A_u(J)$. (Received July 15, 1946.)

264. A. T. Brauer: *Limits for the characteristic roots of a matrix.*

Let $A = (a_{\kappa\lambda})$ be a matrix with real or complex elements. Set $\sum_{\lambda=1}^{n} |a_{\kappa\lambda}| = R_\kappa$, $\sum_{\kappa=1}^{n} |a_{\kappa\lambda}| = T_\lambda$; $\max_{\kappa=1,2,\cdots,n}\{R_\kappa\} = R$, $\max_{\lambda=1,2,\cdots,n}\{T_\lambda\} = T$; $R_\kappa - |a_{\kappa\kappa}| = P_\kappa$, $T_\lambda - |a_{\lambda\lambda}| = Q_\lambda$. It is proved that each characteristic root ω_ν of A lies in at least one of the circles $|z - a_{\kappa\kappa}| \leqq P_\kappa$ and in at least one of the circles $|z - a_{\lambda\lambda}| \leqq Q_\lambda$. It follows that $|\omega_\nu| \leqq \min(R, T)$. This generalizes a result of Frobenius for matrices with posi-