A NOTE ON LINEAR HOMOGENEOUS DIOPHANTINE EQUATIONS

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In this paper the coefficients a_{ij} in the equations

$$a_{i1}x_1 + \cdots + a_{in}x_n = 0 \qquad (i = 1, \cdots, m) \tag{1}$$

are constant rational integers and all letters denote integers. If $m=n-1$ and the rank is $n-1$ then the complete solution in integers is well known. Thus, if E_j is the determinant obtained by deleting the jth column from the matrix of the coefficients, and if $e=(E_1, \cdots, E_n)$, then the solution is

$$x_j = (-1)^j tE_j/e \qquad (j = 1, \cdots, n), \tag{2}$$

in which t is an arbitrary integer.

E. T. Bell recently conjectured that if $m<n-1$ and if the rank r is m then the solution is similarly obtained from the system formed by (1) and the equations

$$\xi_{i1}x_1 + \cdots + \xi_{in}x_n = 0 \qquad (i = 1, \cdots, n-m-1), \tag{3}$$

in which the ξ_{ij} are arbitrary integers. In this paper this conjecture is proved by induction. Since this solution is written down directly from (1) and is fully displayed these results are more usable than those in the literature.[1]

If $r=1$ it can be assumed without limitation that $a_1 \cdots a_n \neq 0$, $(a_1, \cdots, a_n)=1$, and at least one of $x_1, \cdots, x_n$ is not zero. If $n=3$ there are integers $t, y_1, y_2, y_3, d, A_1, A_2, k_1, k_2$ such that

$$x_1 = ty_1, \qquad x_2 = ty_2, \qquad x_3 = ty_3, \qquad (y_1, y_2, y_3) = 1, \tag{4}$$

$$a_1 = dA_1, \qquad a_2 = dA_2, \qquad (A_1, A_2) = 1, \qquad k_1A_2 - k_2A_1 = 1. \tag{5}$$

Since $(d, a_3)=1$ there is an integer s such that

$$y_3 = ds, \qquad A_1y_1 + A_2y_2 + a_3s = 0. \tag{6}$$

Then since $(A_1, A_2)=1$ there is an integer r such that

$$y_1 - a_3k_2s = A_2r, \qquad y_2 + a_3k_1s = -A_1r. \tag{7}$$

These conditions are also sufficient. Hence the complete solution is

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[1] Th. Skolem, *Diophantische Gleichungen*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 5, no. 4, 1938; D. N. Lehmer, Proc. Nat. Acad. Sci. U.S.A. vol. 4 (1919).