

BIBLIOGRAPHY

1. A. Ostrowski, *Über die Bedeutung der Jensenschen Formel für einige Fragen der komplexen Functionentheorie*, Acta Univ. Szeged. vol. 1 (1923) pp. 80–87.
2. H. Milloux, *Le théorème de M. Picard, suites de fonctions holomorphes, fonctions méromorphes et fonctions entières*, J. Math. Pures Appl. (9) vol. 3 (1924) pp. 345–401.
3. R. E. A. C. Paley, *On the lacunary coefficients of power series*, Ann. of Math. vol. 34 (1933) pp. 615–616.
4. R. Nevanlinna, *Le théorème de Picard-Borel et la théorie des fonctions méromorphes*, Paris, 1929, pp. 133 and 138.
5. G. H. Hardy and J. E. Littlewood, *A further note on the converse of Abel's theorem*, Proc. London Math. Soc. (2) vol. 25 (1926) pp. 219–236.
6. A. Zygmund, *On the convergence of lacunary trigonometric series*, Fund. Math. vol. 16 (1930) pp. 90–107.

SWARTHMORE COLLEGE

ON THE (C, 1) SUMMABILITY OF CERTAIN RANDOM SEQUENCES

HERBERT ROBBINS

It is known [1]¹ that if a sequence $\{a_n\}$ ($n=1, 2, \dots$) of real numbers is summable (C, 1) to a value α , and if $\sum a_n^2/n^2 < \infty$, then almost all the subsequences of $\{a_n\}$ are summable (C, 1) to α . It will be shown that this statement continues to hold if “almost all” is replaced by “with probability 1” and “subsequences” by the more general term “product sequences,” the meaning of which will be defined in the next paragraph. The only analytic tool used is the strong law of large numbers [2]: if $\{y_n\}$ is a sequence of independent random variables with expected values $E(y_n) = 0$ and $E(y_n^2) = b_n^2$, for which $\sum b_n^2/n^2 < \infty$, then with probability 1 the sequence $\{y_n\}$ is summable (C, 1) to the value 0.

DEFINITION. Let $\{a_n\}$ be a sequence of constants and let $\{x_n\}$ be a sequence of random variables such that the values of each x_n are non-negative integers. For every n let $k(n)$ be the least positive integer m such that

$$(1) \quad \sum_1^m x_i \geq n,$$

Presented to the Society, September 17, 1945; received by the editors January 23, 1946.

¹ Numbers in brackets refer to references listed at end of paper.