A THEOREM ON ARBITRARY J-FRACTIONS

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1. Introduction. We consider a J-fraction

$$\frac{1}{b_1 + z - \cfrac{a_1^2}{b_2 + z - \cfrac{a_2^2}{b_3 + z - \cdots}}} \quad (a_p \neq 0), \tag{1.1}$$

in which the coefficients a_p and b_p are any complex numbers, the a_p being different from zero, and z is a complex parameter. The system of linear equations

$$- a_{p-1}x_{p-1} + (b_p + z)x_p - a_p x_{p+1} = 0, \quad p = 1, 2, 3, \cdots; a_0 = 1, \tag{1.2}$$

can be solved for $x_2, x_3, x_4, \cdots$ uniquely in terms of arbitrarily chosen initial values x_0 and x_1. We denote by $X_p(z)$ and $Y_p(z)$ the solutions corresponding to $x_0 = -1$, $x_1 = 0$ and $x_0 = 0$, $x_1 = 1$, respectively: $X_0(z) = -1$, $X_1(z) = 0$, $Y_0(z) = 0$, $Y_1(z) = 1$. Then $X_{p+1}(z)/Y_{p+1}(z)$ is the pth approximant of the J-fraction, and we have the determinant formula

$$X_{p+1}(z)Y_p(z) - X_p(z)Y_{p+1}(z) = 1/a_p, \qquad p = 0, 1, 2, \cdots. \tag{1.3}$$

The following theorem holds.

THEOREM OF INVARIABILITY. *If the series*

$$\sum_{p=1}^{\infty} | X_p(z) |^2, \qquad \sum_{p=1}^{\infty} | Y_p(z) |^2 \tag{1.4}$$

converge for a single value of the parameter z, then these series converge uniformly over every bounded domain of z.

This theorem was proved by Hellinger and Wall [3].[1] The uniformity of the convergence was not explicitly mentioned, but is contained

Presented to the Society, September 17, 1945; received by the editors December 26, 1945.

[1] Numbers in brackets refer to the Bibliography at the end of the paper.