

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

222. H. S. M. Coxeter: *A simple proof of the eight square theorem.*

It is shown, by direct comparison of terms, that the conjugates of Cayley numbers have the properties $\overline{ab} = \overline{b} \overline{a}$ and $\overline{a} a \cdot b = \overline{a} \cdot ab$. Defining $a^{-1} = (Na)^{-1} \overline{a}$, it is deduced that $b = a^{-1} \cdot ab$ and similarly $ab \cdot b^{-1} = a$. As Ruth Moufang remarked, these relations imply $b^{-1} a^{-1} = (ab)^{-1}$. One now has $N(ab) = \overline{a} \overline{b} \cdot ab = \overline{b} \overline{a} \cdot ab = Nb \cdot Na \cdot b^{-1} a^{-1} \cdot ab = Na \cdot Nb$. (Cf. Dickson, *Ann. of Math.* (2) vol. 20 (1919) p. 164.) (Received May 29, 1946.)

223. H. S. M. Coxeter: *Integral Cayley numbers.*

The Cayley numbers $a_0 + a_1 i + a_2 j + a_3 k + (a_4 + a_5 i + a_6 j + a_7 k) h$, where $h = 2^{-1}(i + j + k + e)$ and the a 's are arbitrary integers, are shown to be a set of integral elements in the sense of Dickson (*Algebras and their arithmetics*, Chicago, 1923, pp. 141–142). The integral Cayley numbers of norm 1, 2 and 4 are represented by the vertices of the eight-dimensional polytopes 4_{21} , 2_{41} and 1_{42} . (Received May 29, 1946.)

224. Roy Dubisch: *On the direct product of rational generalized quaternion algebras.*

Utilizing the results of Albert (*Bull. Amer. Math. Soc.* vol. 40 (1934) pp. 164–176) on the integral domains of rational generalized quaternion algebras and the results of Latimer (*Duke Math. J.* vol. 1 (1935) pp. 433–435) on the fundamental number of a rational generalized quaternion algebra, the author proves that any finite number of such algebras $\mathfrak{Q}_1, \dots, \mathfrak{Q}_n$ contain a common quadratic subfield \mathfrak{B} and $\mathfrak{Q}_i = (\mathfrak{B}, S, -d_i)$ ($i = 1, \dots, n$) where d_i is the fundamental number of \mathfrak{Q}_i . Then $\mathfrak{Q}_1 \times \dots \times \mathfrak{Q}_n \sim \mathfrak{B} = (\mathfrak{B}, S, \pm \Pi d_i)$. (Received May 27, 1946.)

225. Orrin Frink: *Complemented modular lattices and projective spaces of infinite dimension.*

Garrett Birkhoff (*Ann. of Math.* vol. 36 (1935) pp. 743–748) showed that every complemented modular lattice of finite dimension is the direct union of the lattices of subspaces of projective geometries. In this paper complemented modular lattices in general, without restriction on the dimension, are characterized as subdirect unions of the subspace lattices of projective planes and irreducible projective coordinate spaces of possibly infinite dimension. It is shown that every complemented modular lattice determines a unique projective space whose points are the maximal dual ideals