BOOK REVIEW

Jacobian elliptic functions. By E. H. Neville. Oxford University Press, 1944. 16+331 pp. $7.50.

In the development of the theory of elliptic functions it is shown at an early stage that, as far as singularities are concerned, the simplest elliptic functions other than constants are those of order 2. This naturally leads to the classification of elliptic functions of order 2 into those with one double pole (of zero residue) and those with two simple poles in the parallelogram of periods. The Weierstrassian theory of doubly periodic functions, the theory which is still frequently included in a first course of complex variables, starts with a function $\wp(z)$ of the first kind which has the double pole at the origin. By using Liouville's general theorems, elliptic functions with singularities, arbitrary within the permissible limits, are constructed.

The Jacobian theory starts out basically with functions of the second kind, and it is Professor Neville's merit to lay particular stress on this purely function-theoretic classification. Historically, the Jacobian functions arose in connection with the inversion of the Legendre integral

$$u = \int_0^{\phi} (1 - k^2 \sin^2 \theta)^{-1/2} d\theta$$

as $\operatorname{sn} u = \sin \phi$, $\operatorname{cn} u = \cos \phi$, $\operatorname{dn} u = d\phi/du$. While this manner of introduction of the Jacobian functions is understandable enough in view of the fact that the study of elliptic functions was prompted by the occurrence in physical and geometrical problems of certain integrals which could not be evaluated by elementary functions, it certainly lends a character of arbitrariness to the selection of the standard functions of the theory. The other procedure followed in the past has been to postpone the theory of the functions $\operatorname{sn} u$, $\operatorname{cn} u$, $\operatorname{dn} u$ until the study of theta functions. The function $\operatorname{sn} u$ is then introduced by means of the relation

$$\operatorname{sn} u = \frac{\theta_3}{\theta_2} \cdot \frac{\theta_1(u/\theta_3^2)}{\theta_4(u/\theta_3^2)},$$

which is hardly calculated to inspire a reader with a sense of the importance and necessity of studying Jacobian functions.

In view of this fact, the author's procedure certainly deserves