THE TRIANGULATION PROBLEM AND ITS ROLE IN ANALYSIS

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1. **Introduction.** The triangulation problem is fundamental in the topology of manifolds and is closely related to certain methods employed in analysis. The results to date are, perhaps, of greater interest from the viewpoint of connections between topology and differential geometry (or other branches of analysis) than from a purely topological viewpoint. Most of the published work on this problem has appeared during approximately the last fifteen years, save for well known results in two dimensions. A brief discussion of a 2-dimensional result and its role in proving a theorem of analysis (adapted from Osgood's *Funktionentheorie* [31][1]) may throw light on the problem and a certain class of applications.

Let B denote a simple closed curve in the (x, y)-plane, and let R denote its interior. It will be assumed that B is differentiable in the sense that some neighborhood of any point on B can be represented by giving y (or x) as a single-valued function of x (or y) with a continuous first derivative. Let the entire (x, y)-plane be subdivided into squares by the lines

$$x = m\delta, \quad y = n\delta, \qquad m, n = 0, \pm 1, \pm 2, \cdots, \tag{1.1}$$

where δ is a positive number so small that a circle of radius 3δ about any point of B cuts from B a single arc, any two tangents to which form an angle less than $\pi/6$. If, in or on the boundary of one of the squares determined by (1.1), B is (1) parallel at some point to an axial direction and (2) meets an edge, β, parallel to that same direction, then let the two squares incident with β be amalgamated into a single rectangular region. The amalgamated region cuts from B a single arc with end points on the sides perpendicular to β. This arc divides the rectangle into two parts, one in R and one outside. The part inside R will be a 2-cell of the subdivision of $(R+B)$. Each of the squares which meets $(R+B)$ and is not involved in such an amalgamation has in its interior just a 2-cell of R, to be reckoned as a 2-cell of the subdivision. The 1-cells of the subdivision consist of (1) the edges entirely in R of the squares determined by (1.1), (2) the

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[1] Numbers in brackets refer to the Bibliography at the end of the paper.