TWO NONEXISTENCE THEOREMS ON PARTITIONS

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The theory of partitions contains a number of theorems which assert that the number of partitions of a given number into parts subjected to a certain restriction is the same as the number of partitions restricted in some other way. A common type of restricted partition is one in which all parts are distinct. We have for example the famous theorem of Euler[1] (1748):

EULER'S THEOREM. *The number of partitions of n into distinct parts is the same as the number of partitions of n into odd parts.*

The notion of distinctness of parts may be altered in two directions. One may relax it to some extent and admit partitions in which no part is repeated more than a given number of times. In this case we have the beautiful theorem of Glaisher[2] (1883).

GLAISHER'S THEOREM. *The number of partitions of n in which no part is repeated more than $r-1$ times is the same as the number of partitions of n into parts not divisible by r.*

This theorem obviously becomes Euler's theorem when $r=2$.

On the other hand the notion of distinctness may be further restricted so as to include only those partitions in which the parts differ by d or more. For $d=0$, we have completely unrestricted partitions. For $d=2$ we have a celebrated and difficult theorem discovered independently by Rogers[3] (1894), Schur[4] (1917) and Ramanujan[5] (1919).

ROGERS' THEOREM. *The number of partitions of n into parts differing by 2 or more is the same as the number of partitions of n into parts taken from the set* $1, 4, 6, 9, 11, 14, 16, \cdots, (5k+1, 4), \cdots$.

Attempting to go further in this direction, Schur[6] later (1926) proved the following theorem:

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[1] L. Euler, *Introductio analysin infinitorum*, vol. 1, Lausanne, 1748, pp. 253–275.

[2] J. W. L. Glaisher, Messenger of Mathematics vol. 12 (1883) pp. 158–170.

[3] L. J. Rogers, Proc. London Math. Soc. (1) vol. 25 (1894) pp. 328–329.

[4] I. Schur, Akademie der Wissenschaften, Berlin, Sitzungsberichte (1917) pp. 302–321.

[5] S. Ramanujan, Proc. Cambridge Philos. Soc. vol. 19 (1919) pp. 211–216.

[6] I. Schur, Akademie der Wissenschaften, Berlin, Sitzungsberichte (1926) pp. 488–495. See also W. Gleissberg, Math. Zeit. vol. 28 (1928) pp. 372–382.