A NOTE ON POINTWISE NONWANDERING TRANSFORMATIONS

W. H. GOTTSCHALK

Let X be a T_1-space and let f be a continuous transformation of X into X In the terminology of G. D. Birkhoff [1, p. 191],[1] a point x of X is said to be *nonwandering* under f provided that to each neighborhood U of x there correspond infinitely many positive integers n for which $U \cap f^n(U) \neq \emptyset$; also, the transformation f is said to be *pointwise nonwandering* provided that each point of X is nonwandering under f.

THEOREM 1. *If f is pointwise nonwandering, then so also is f^k for every positive integer k.*

PROOF. (We make use of a technique employed by Erdös and Stone [2, pp. 126–127].) Suppose k is a positive integer, $x_0 \in X$, and U_0 is a neighborhood (= open neighborhood) of x_0. Let n_1 be a positive integer for which $U_0 \cap f^{n_1}(U_0) \neq \emptyset$. Choose $x_1 \in U_0$ so that $f^{n_1}(x_1) \in U_0$ and a neighborhood U_1 of x_1 so that $U_1 \subset U_0$ and $f^{n_1}(U_1) \subset U_0$. Let n_2 be a positive integer for which $U_1 \cap f^{n_2}(U_1) \neq \emptyset$. Choose $x_2 \in U_1$ so that $f^{n_2}(x_2) \in U_1$ and a neighborhood U_2 of x_2 so that $U_2 \subset U_1$ and $f^{n_2}(U_2) \subset U_1$. Continuing this process, we obtain a sequence $\{n_i\}$ of positive integers and a sequence $\{U_i\}$ of neighborhoods so that $U_i \subset U_{i-1}$ and $f^{n_i}(U_i) \subset U_{i-1}$ $(i=1, 2, \cdots)$. Let r_i denote the integer for which $1 \leqq r_i \leqq k$ and $n_i \equiv r_i \bmod k$. Infinitely many of the r_i are equal to some integer, say r. We may suppose $r_i = r$, $U_i \subset U_{i-1}$ and $f^{n_i}(U_i) \subset U_{i-1}$ $(i=1, 2, \cdots)$. Choose an arbitrary positive integer p. Define $n = \sum_{i=1}^{pk} n_i$. Now $n \equiv 0 \bmod k$. Choose $x \in U_{pk}$. Clearly, $x \in U_0$ and $f^n(x) \in U_0$. Hence, $U_0 \cap f^n(U_0) \neq \emptyset$. Since $n \geqq p$, the proof is completed.

LEMMA 1. *If $f(X) = X$ is a homeomorphism, if A and B are closed connected sets for which $A \cup B = X$, $A \cap B = x \in X$ and $A \cap f(A) \neq \emptyset \neq B \cap f(B)$, and if x is nonwandering, then x is fixed.*

PROOF. Assume x is not fixed. We may suppose that $f(x) \in B$. Now $x \notin f^{-1}(A)$ for in the contrary case $f(x) \in A \cap B = x$. The set $f(A)$ is connected and intersects both A and B. Hence, $x \in f(A)$. There exists a neighborhood U of x such that $U \cap f^{-1}(A) = \emptyset$ and such that

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[1] Numbers in brackets refer to the bibliography at the end of the paper.