GENERALIZATIONS OF TWO THEOREMS OF JANISZEWSKI. II

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The purpose of this note is to strengthen Theorems 5 and 6 of [1][1] and to make corrections regarding assumptions of compactness in that paper. The following theorems hold in the plane.

THEOREM 1. *If neither of the domains D_1, D_2 separates the point A from the point B, the boundary of D_1 is compact and the common part of D_2 and each component of D_1 is connected or does not exist, then D_1+D_2 does not separate A from B.*

PROOF. Assume that D_1+D_2 separates A from B. Considering there to be a point P at infinity, we find that D_1+D_2+P contains a simple closed curve J separating A from B. Let d_2 be a component of D_2 intersecting J. We find [1, Theorem 4] that $J-J\cdot d_2$ contains a continuum M cutting A from B in the complement of d_2 and such that any open arc of J containing M separates A from B in the complement of d_2. Let d_1 be a component of D_1 covering a point of M on the boundary of d_2. Now d_1 covers M or else it would intersect two components of D_2. But by Theorem 5 of [1], d_1+d_2 does not separate A from B.

Instead of assuming that the boundary of D_1 is compact, we could assume that the part of D_1 in the complement of D_2 is compact.

THEOREM 2. *If neither of the domains D_1, D_2 cuts the point A from the point B, the boundary of D_1 is compact and the common part of D_2 and each component of D_1 is connected or does not exist, then D_1+D_2 does not cut A from B.*

PROOF. Let C_i $(i=1, 2)$ be the component of the complement of D_i containing $A+B$, let D_i' be the complement of C_i and let D_2'' be the sum of all components of D_2' that are not covered by D_1'. Neither D_1' nor D_2'' separates the plane. The boundary of D_1' is a subset of the boundary of D_1 and is therefore compact. If d' is a component of D_1', we shall show that $d'\cdot D_2''$ is connected or does not exist. It will follow from Theorem 1 that $D_1'+D_2''$ does not separate the plane. Hence, its complement is a continuum containing $A+B$ and its subset D_1+D_2 does not cut A from B.

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[1] Number in brackets refers to the reference cited at the end of the paper.