

shown that, in an important sense, the only admissible measure of fluctuation, except for a constant factor, is the variance $f(a_1, \dots, a_n) = \sum_{i=1}^n (a_i - n^{-1} \sum_{j=1}^n a_j)^2$. The method of maximum likelihood is applied to special functions to obtain tests for significant differences. These tests have applications to industrial problems. (Received March 22, 1946.)

198. Isaac Opatowski: *Average duration of transition in Markoff chains.*

Consider a chain of transitions ($i \rightarrow i+1$), ($i+1 \rightarrow i$), where $i=0, 1, \dots, n-1$. Let the usual conditional probabilities of these transitions within any time Δt be respectively $k_{i+1}\Delta t + o(\Delta t)$ and $g_i\Delta t + o(\Delta t)$, where the k_i 's and g_i 's are constant. Let the probability of any other transition during Δt be $o(\Delta t)$. The probability $P(t)$ of a transition ($0 \rightarrow n$) within a time t is an increasing function of t and $P(\infty) = \prod_{i=1}^{n-1} k_i / K_i$, where $s = -K_i$ are the n roots of the secular equation $\|a_{r,c}\|/s = 0$ defined by $a_{r,r} = s + k_{r+1} + g_{r-1}$, $a_{r,r-1} = k_r$, $a_{r,r+1} = g_r$, $a_{r,c} = 0$ for $|r-c| > 1$; $r, c = 0, 1, \dots, n$; $g_{-1} = 0$ (Bulletin of Mathematical Biophysics vol. 7 (1945) pp. 170-177). If $g_{n-1} = 0$ then $P(\infty) = 1$, if $g_{n-1} \neq 0$ then $P(\infty) < 1$. Consequently, the average duration of the transition ($0 \rightarrow n$) is $E = \int_0^{P(\infty)} t dP / P(\infty)$. Its explicit expression is a simple symmetric function of the k_i 's and K_i 's. If all the g_i 's are zero, $E = \sum_{i=1}^{n-1} k_i^{-1}$. By using the results of a previous paper (Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) pp. 411-414) simple expressions of E are obtained also in the cases in which the k_i 's and g_i 's are independent of i . The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 19, 1946.)

199. Isaac Opatowski: *Markoff chains with variable intensities: average duration of transition.*

Consider a simple Markoff chain. Let $k_{i+1}\Delta t + o(\Delta t)$ be the conditional probability of a transition ($i \rightarrow i+1$) within any time Δt , where $i=0, 1, \dots, n-1$ and $k_i = F(i)f(t)$. It is known that by changing t into a new time variable $T = \int_0^t f(t) dt$, the present chain may be treated as if its intensities k_i were constant and equal to $F(i)$. Let $t = \sum_{m=1}^n c_m T^m$ be a polynomial in T . Let $P(t)$ be the probability of a transition ($0 \rightarrow n$) within t . It is shown that $\int_0^t t dP$, the average duration of a transition ($0 \rightarrow n$), equals $\sum_{m=1}^n m! c_m h_m$, where h_m is the complete homogeneous symmetric function of degree m of $1/F(1), 1/F(2), \dots, 1/F(n)$. This formula is obtained by using a previous result on the moments of Markoff chains (Proc. Nat. Acad. Sci. U.S.A. vol. 28 (1942) pp. 83-88). The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 20, 1946.)

TOPOLOGY

200. E. E. Floyd: *On the extensions of homeomorphisms on the interior of a two cell.*

Let $f(I) = R$ be a homeomorphism of the interior I of a two cell with boundary C onto a bounded plane region R . It is shown that if f is extensible to \bar{I} , then the extension is non-alternating on the boundary C . A condition is also derived which is equivalent to the existence of an extension g of f , where $g(\bar{I}) = \bar{R}$, $g = f$ on I , and g is light and non-alternating on C . This is applied to conformal maps, yielding the following theorem: let $f(I) = R$ be a 1-1, conformal map of the interior I of the unit circle onto a