

others, sought a new logic more adequate than the traditional one to modern science and practice. Prior to Frege or Russell, Hegel possessed logistic intent which was determining factor in development of his logic. He was familiar with historic roots of modern symbolic logic—with work of Lully, Bruno, Leibniz, Ploucquet, Euler, and Bardili—rejecting their primitive efforts as inadequate. He knew the Sancho Panza dilemma, recognized by Church as closely related to Russell's paradox, and even proposes solution resembling theory of types. His denial of so-called "laws of thought" antedates denial of law  $A \cdot A = A^2$  in Boole's logical algebra, of parallel-axiom in non-Euclidean geometry, of commutative law in quaternion theory. When mathematicians and mathematical philosophers were seeking to avoid the infinite, Hegel restored it, with new interpretation, to a place of central importance in foundations of analysis. He speaks of infinite as involving equality of whole and part, distinguishes "bad" infinite (Cantor's variable finite) from "good" infinite (recognized as reflexive) and objects to the phrase "and so on to infinity," shown eliminable by Frege. Once influential, he was known to Boole, DeMorgan, Bolzano, Pierce, G. Cantor, and Russell, while Frege refers to Fischer's Hegelian logic. (Received March 23, 1946.)

#### STATISTICS AND PROBABILITY

195. Will Feller: *A limit theorem for random variables with infinite moments.*

Let  $\{X_k\}$  be an arbitrary sequence of mutually independent random variables with the same distribution function  $V(x)$ . It is assumed that some moment of order less than two is infinite; the first moment may be infinite, but if it is finite it should be normed to zero. Let  $S_n = X_1 + \dots + X_n$  and let  $\{a_n\}$  be a monotonic positive numerical sequence. It is shown that the probability that the inequality  $|S_n| > a_n$  takes place for infinitely many  $n$  is the same as the probability that  $|X_n| > a_n$  for infinitely many  $n$ ; it is one or zero according as the series  $\sum \{V(-a_n) + 1 - V(a_n)\}$  diverges or converges. (Received March 21, 1946.)

196. Will Feller: *The law of the iterated logarithm for identically distributed random variables.*

Let  $\{X_n\}$  be a sequence of mutually independent random variables with the same distribution function  $V(x)$  with vanishing first moment and unit variance. Suppose that (\*)  $\int_{|t|>x} t^2 dV(t) = O((\log \log x)^{-1})$ , and let  $\{\phi_n\}$  be an arbitrary monotonic sequence,  $\phi_n > 0$ . The probability that the inequality  $X_1 + \dots + X_n > n^{1/2} \phi_n$  will be satisfied for infinitely many  $n$  is shown to be zero or one according as the series  $\sum \phi_n n^{-1} \exp(-\phi_n^2/2)$  converges or diverges. The condition (\*) is in a certain sense the best possible. If it is not satisfied, the above exact analogue to the strict law of the iterated logarithm does not hold, but slightly more complicated necessary and sufficient conditions are given in the paper. (Received March 21, 1946.)

197. Casper Goffman: *Measures of fluctuation of a variable mean.* Preliminary report.

Suppose a finite order of random variables  $x_1, \dots, x_n$  is given, all normally distributed, with the same known standard deviation, but with unknown means  $a_1, \dots, a_n$ , not necessarily alike. A measure of fluctuation of the means is defined as a function  $f(a_1, \dots, a_n)$  such that (1) for every real  $h$ ,  $f(a_1+h, \dots, a_n+h) = f(a_1, \dots, a_n)$ , and (2) for every real  $c$ ,  $f(ca_1, \dots, ca_n) = c^2 f(a_1, \dots, a_n)$ . It is