

$1 \leq r < m_1$ , in such a way that there are  $m-r$  linearly independent omniconjugate directions at  $P$  with respect to the  $V_{m+r}$ . (The local  $R_{m+r}$  of the  $V_{m+r}$  at  $P$  is contained within the local  $R_{m+m_1}$ .) If there is a second normal space of  $m_2$  dimensions, then  $m_2 \leq s(s+1)(s+2)/6 + t(t+1)/2$ , where  $s$  and  $t$  are integers determined by  $s(s+1)/2 \leq m_1 \leq (s+1)(s+2)/2$ ,  $t = m_1 - s(s+1)/2$ . Similar statements can be made about the vanishing of the third and other higher normal spaces. (Received March 19, 1946.)

192. Y. C. Wong: *Contributions to the theory of surfaces in a 4-space of constant curvature.*

A Riemannian 4-space of constant curvature and a surface in it are denoted by  $S_4$  and  $V_2$ , respectively. The method of studying  $V_2$  in  $S_4$  in this paper is invariant and is similar to those of G. Ricci (*Lezioni sulla teoria della superficie*, Verona-Padova, 1898) and W. Graustein (Bull. Amer. Math. Soc. vol. 36 (1930)) for their studies of surfaces in a Euclidean 3-space. In essence, the method consists of setting up a suitable system of invariant fundamental equations for a  $V_2$  in  $S_4$ , and expressing the required imbedding conditions of  $V_2$  in  $S_4$  in terms of the intrinsic properties of  $V_2$ . Curvature properties, especially those about the curvature conic, of a general  $V_2$  in  $S_4$  are first discussed. Then the  $V_2$ 's whose curvature conic is of certain particular nature are studied. These include the minimal  $V_2$ , with the  $R$ -surface of K. Kommerell (Math. Ann. vol. 60 (1905)) as a special case, ruled  $V_2$ , and  $V_2$  with an orthogonal net of Voss. The paper concludes with a complete determination of those  $V_2$ 's in  $S_4$  whose first fundamental form and one of whose second fundamental forms are respectively identical with the first and second fundamental forms of a surface in a 3-space of constant curvature. (Received March 11, 1946.)

193. Y. C. Wong: *Scale hypersurfaces for conformal-Euclidean space.*

This paper contains generalizations to  $n$ -space of some of the results obtained recently by E. Kasner and J. DeCicco (Amer. J. Math. vol. 67 (1945)) for the scale curves in conformal maps of a surface on a plane. The fundamental form  $ds^2 = e^{2\sigma}(dx_1^2 + \dots + dx_n^2)$ , with  $\sigma = \sigma(x_1, \dots, x_n)$ , represents a conformal-Euclidean  $n$ -space  $C_n$ , conformally mappable on the Euclidean  $n$ -space  $R_n$  with rectangular Cartesian coordinates  $x_1, \dots, x_n$ . The hypersurfaces  $\sigma = \text{constant}$  in  $R_n$  are the scale hypersurfaces in the mapping of  $C_n$  on  $R_n$ , and any simple family of hypersurfaces in  $R_n$  is called quasi-isothermal if it represents the scale hypersurfaces of a conformal mapping of some  $C_n$  on  $R_n$  such that the scalar curvature of  $C_n$  is constant over each of the scale hypersurfaces. A few theorems are proved concerning the cases when a family of quasi-isothermal hypersurfaces is a family of (a)  $\infty^1$  hyperplanes, (b)  $\infty^1$  generalized cylinders of rotation. This subject is closely connected with the subject of the isoparametric hypersurfaces of T. Levi-Civita and B. Segre (Rendiconti della Reale Accademia Nazionale dei Lincei (6) vol. 26 (1937), vol. 27 (1938)) and incidentally connected with that of the subprojective Riemannian space of B. Kagan and H. Schapiro (*Abhandlung des Seminars für Vektor- und Tensoranalysis*, vol. 1, 1933). (Received March 11, 1946.)

#### LOGIC AND FOUNDATIONS

194. Ira Rosenbaum: *Hegel, mathematical logic, and the foundations of mathematics.*

Hegel, like Boole, DeMorgan, Pierce, Frege, Peano, Russell, Whitehead and