PHYSICS AND THE WAVE EQUATION

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Often in the history of physics a guiding line of mathematical thought has permeated the whole of the science for years, tying together apparently unrelated branches of the subject, contributing to the unity of physics, but at the same time stimulating philosophical thought, and focussing attention on a branch of mathematics, and leading to its development. The Newtonian mechanics was such a guiding principle. From Kepler through Newton and up into the nineteenth century, more and more of physics was explicable in mechanical form. Even philosophy and economics and history felt the impact of rationalism. Mathematics felt the tide; calculus and the theory of ordinary differential equations grew up under the impetus of the physicist, who needed the mathematical methods to explain his physical facts. A second guiding principle was the variation principle. D'Alembert, Lagrange, Hamilton expressed the laws of mechanics in variational form. As time went on, more and more branches of physics could be formulated in similar language. We had not merely the principle of least action in mechanics, but Fermat's principle in optics, and variational formulations of electromagnetic theory. Here again there were impacts on both philosophy and mathematics. The philosophers grasped at the principle of least action as a proof of the existence of the deity, who used the simplest and most effective means to accomplish his purposes. The mathematicians were led to the development of the calculus of variations, and to such related fields as the theory of continuous groups and of contact transformations. Several similar developments have come since that time; two conspicuous ones are statistics, as seen in statistical mechanics, in the philosophical ideas associated with the second law of thermodynamics, and in the mathematical development of the theory of statistics; and relativity, with its obvious philosophical accompaniments, and its relation to the theories of the absolute differential calculus and tensor analysis. In all of these cases, I believe one could make out a case for the thesis that each succeeding line of thought in physics enriched and supplemented, but never supplanted, those which had gone before; that the philosophical applications were in general superficial and ephemeral, the embodiment not of fundamental truth but of the

The nineteenth Josiah Willard Gibbs lecture delivered at Chicago, Illinois, November 23, 1945, under the auspices of the American Mathematical Society and the Mathematical Association of America: received by the editors December 6, 1945.