CONCERNING THE SEPARABILITY OF CERTAIN LOCALLY CONNECTED METRIC SPACES

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If a connected metric space S is locally separable, then S is separable.[1] If a connected, *locally connected*, metric space S is locally *peripherally* separable, then S is separable.[2] Furthermore if a connected, locally connected, *complete* metric space S satisfies certain "flatness" conditions, it is known to be separable.[3] These "flatness" conditions are rather strong and involve both im kleinen and im grossen properties, which makes application rather awkward in some cases. If, however, this space S contains no skew curve[4] of type 1, then S has a certain amount of "flatness," but not quite enough to imply separability as can be seen from the following example. Let S consist of the points of the 2-sphere, distance being redefined as follows: (1) if the points X and Y of S lie on the same great circle through the poles, then $d(X, Y)$ is the ordinary distance on the sphere but (2) if the points lie on different great circles through the poles, then $d(X, Y)$ is the sum of the ordinary distances from each point to the same pole, using the pole which gives the smaller sum. The space S is a connected, locally connected, complete metric space which contains no skew curve of type 1 but S is not separable. Furthermore, S contains no cut point. However, if this last condition is strengthened slightly, separability follows as is seen in the following theorem.

THEOREM 1. *Let S denote a locally connected, complete metric space such that no pair of points cuts S. If S contains no skew curve of type* 1, *then S is separable.*

PROOF. Suppose, on the contrary, that S is not separable. Let T_0

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[1] Paul Alexandroff, *Über die Metrization der im kleinen kompakten topologischen Räume*, Math. Ann. vol. 92 (1924) pp. 294–301. Also W. Sierpinski, *Sur les espaces métriques localement separables*, Fund. Math. vol. 21 (1933) pp. 107–113.

[2] F. B. Jones, *A theorem concerning locally peripherally separable spaces*, Bull. Amer. Math. Soc. vol. 41 (1935) pp. 437–439.

[3] F. B. Jones, *Concerning certain topologically flat spaces*, Trans. Amer. Math Soc. vol. 42 (1937) pp. 53–93, Theorem 31. Also F. B. Jones, Bull. Amer. Math. Soc. Abstract 47-1-93.

[4] Kuratowski in his paper, *Sur le problème des courbes gauches en Topologie*, Fund. Math. vol. 15 (1930) pp. 271–283, defined two "skew curves." One of type 1 is topologically equivalent to the sum of three simple triods each two of which intersect precisely at their end points.