TWO BRIEF FORMULATIONS OF BOOLEAN ALGEBRA

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This paper presents two new formulations of Boolean algebra which appear to have some direct interest on their own account, and which further take a place among the most economical versions, while also exhibiting more intuitive clarity than other versions of a similar degree of economy.[1] As is to some extent customary in short expositions of this kind, I am taking for granted without formal presentation: (1) that the system contains more than one element; (2) that it is closed with respect to such operations as appear explicitly in the axioms; (3) that I may employ an identity (or equality) relation without formal statement of its properties; (4) that in addition I may utilize recognized principles of logic (in fact only an ordinary or elementary logic). With a view to comparisons I shall say that what I am presenting are the "transformation axioms." The variables x, y, z, $\cdots$ will represent elements. No other symbolism is required for features of the system except an accent $'$ and juxtaposition for an undefined singulary and binary operation respectively. A symbol for the class of elements is not introduced, because "formation" rules (for example, closure) are left informal. But certain further symbols are used in the formalism (and regarded as taken over from ordinary logic) to enable us to make our statements about the elements and operations (functions) of the system, either as axioms, theorems, or steps in proofs; these further symbols are an identity or equality sign $=$ (the only relation sign employed), and signs serving as connectives between our statements, namely & "and, "$\rightarrow$ "only if" or "if . . . then . . . ," and $\leftrightarrows$ "if and only if."[2] The last-named and the equality sign are the only ones appearing in axioms. Parentheses are used in customary ways. In the derivation of theorems indication is made of axioms and theorems used, except that Axioms II and III (associativity and commutativity) may sometimes be brought into play without explicit mention, and likewise certain frequently em-

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[1] Comparisons as to economy are intended to be limited to axiomatizations in mathematical (not metamathematical) language. The vague expression "intuitive clarity" here means that, in some familiar interpretation, the postulates adopted seem readily intelligible and plausible for a mind of limited mathematical experience.

[2] The "&" between statements is, of course, not to be confused with the "and" between elements, expressed by mere juxtaposition when we choose to interpret juxtaposition in the "and" rather than in the "or" manner (two ways in which it is feasible to interpret juxtaposition).