

conjugates of α is in \mathfrak{P}_i , $i=1, \dots, m$; hence neither is any power of their product. Some such power, however, is in \mathfrak{R} , hence in $\mathfrak{P} \cap \mathfrak{R} \subset \mathfrak{P}_1$. This is a contradiction and completes the proof.

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NOTE ON AN ASYMMETRIC DIOPHANTINE APPROXIMATION

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1. **Introduction.** In a recent paper B. Segre [1]¹ introduced a new type of Diophantine approximation which he called *asymmetric*, since the intervals of approximation are divided into two partial intervals which are in an arbitrarily given ratio. His main result is the following theorem [1, p. 357]:

THEOREM 1. *Every irrational θ has an infinity of rational approximations x/y such that*

$$(1) \quad \frac{-1}{y^2(1+4\tau)^{1/2}} < \frac{x}{y} - \theta < \frac{\tau}{y^2(1+4\tau)^{1/2}} \quad (y > 0),$$

where τ is any given non-negative real number.

This theorem is classic for $\tau=0$, cf. [2, p. 139], and for $\tau=1$ it reduces to the fundamental result due to Hurwitz [2, p. 163]. No other particular cases of the theorem seem to be known.

Segre's proof of (1) is geometrical. The purpose of this note is to show that when $\tau \geq 1$ it is possible to give a very simple arithmetical proof. The method is a generalization of that used by Khintchine [3] for the special case when $\tau=1$.

2. **Proof of Theorem 1.** We suppose that θ is irrational and that $0 < \theta < 1$. For an arbitrary positive integer n form the Farey series² of order n , that is, the ascending series of irreducible fractions between 0 and 1 whose denominators do not exceed n . Let a/b and a'/b' be the two successive terms of this series which satisfy the inequalities $a/b < \theta < a'/b'$. We distinguish two cases.

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¹ Numbers in brackets refer to the references.

² See Hardy and Wright [2, p. 23].