A NOTE ON AXIOMATIC CHARACTERIZATION OF FIELDS

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Since publication of our paper, *Axiomatic characterization of fields by the product formula for valuations*,[1] we have found that the fields of class field theory can be characterized by somewhat weaker axioms; we can drop the assumption, in Axiom 1, that $|\alpha|_{\mathfrak{p}}=1$ for all but a finite number of $\mathfrak{p}$, replacing it by the assumption that the product of all valuations converges absolutely to the limit 1 for all α.

Our original proof can be adapted to the new axiom with a few modifications, which we shall describe here. In §2, we keep Axiom 1 for reference and introduce:

AXIOM 1*. *There is a set $\mathfrak{M}$ of prime divisors $\mathfrak{p}$ and a fixed set of valuations $|\ \ |_{\mathfrak{p}}$, one for each $\mathfrak{p}\in\mathfrak{M}$, such that, for every $\alpha\neq 0$ of k, the product $\prod_{\mathfrak{p}}|\alpha|_{\mathfrak{p}}$ converges absolutely to the limit* 1. (*That is, the series $\sum_{\mathfrak{p}}\log|\alpha|_{\mathfrak{p}}$ converges absolutely to* 0.)

We must then omit the statement that there are only a finite number of archimedean primes, since this does not follow immediately from 1*; instead of it, we use the fact that $\sum_{\mathfrak{p}_\infty}\rho(\mathfrak{p}_\infty)$ and $\sum_{\mathfrak{p}_\infty}\lambda(\mathfrak{p}_\infty)$ converge absolutely. These quantities are defined on p. 480; the convergence follows from the fact that the product over all $\mathfrak{p}_\infty$ of $|1+1|_{\mathfrak{p}_\infty}$ must converge absolutely. Also, we must temporarily broaden the definition of "parallelotope" so as to permit a parallelotope to be defined by any valuation vector $\mathfrak{a}$ for which $\prod_{\mathfrak{p}}|\mathfrak{a}|_{\mathfrak{p}}$ converges absolutely (rather than restricting $\mathfrak{a}$ to be an idèle). In the statement of Axiom 2 we must replace "Axiom 1" by "Axiom 1*," Theorem 2, however, is left unchanged, together with Lemmas 4, 5, and 6, which are needed only to prove it; this theorem shows that the fields of class field theory really satisfy Axiom 1, so that at the end of the whole proof we shall find that Axiom 1 is a consequence of Axioms 1* and 2.

In §3, k is assumed to be any field for which Axioms 1* and 2 hold. Lemma 8 holds under assumption of Axiom 1*, for our slightly more general parallelotopes; in its proof we have only to note, in case of archimedean primes, that the product $\prod_{\mathfrak{p}_\infty}4^{\rho(\mathfrak{p}_\infty)}$ converges absolutely. In Lemma 9, property 2 must be replaced by:

2*. $|\alpha|_{\mathfrak{p}_\infty}\leqq B_{\mathfrak{p}_\infty}|y|_{\mathfrak{p}_\infty}$, *with a set of constants $B_{\mathfrak{p}_\infty}$ for which $\prod_{\mathfrak{p}_\infty}B_{\mathfrak{p}_\infty}$ converges absolutely.*

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[1] Bull. Amer. Math. Soc. vol. 51 (1945) pp. 469–492.