BOOK REVIEWS

The theory of rings. By Nathan Jacobson. (Mathematical Surveys, no. 2.) New York, American Mathematical Society, 1943. 150 pp. $2.25.

In recent years the theory of rings has been one of the centers of most vigorous mathematical life. Although originally an outgrowth of the theory of algebras, it has made itself completely independent of its origin. This was necessitated by two considerations. Firstly, it appeared that the special hypotheses inherent to the theory of algebras were not needed for the greater part of the theory of rings. The latter theory, thus stripped of unnecessary encumbrances, became more general and at the same time simpler, clearer and more elegant. Secondly, there are some important applications of the theory of rings that do not fit into the framework of the theory of algebras, such as the applications to the rings of endomorphisms of abelian operator groups.

In the book under consideration a comprehensive account is given of the theory of rings. In view of the fact that mathematicians from all over the world have contributed toward the growth of the theory and that their results are scattered over all the international mathematical periodicals, the collection of the material is by itself no mean task (Jacobson's bibliography covers eight pages of fine print). But it is even more important and difficult to obtain a unified treatment of such a host of different methods and points of view. In this Jacobson has succeeded admirably by means of the methodological principle which he uses. The knowledge obtained in the abstract theory of rings is first applied to the study of abelian operator groups over rings (Emmy Noether's representation moduli). The structural theory of abelian operator groups is next used for an investigation of their rings of endomorphisms. The cycle is closed by means of the following theorem which permits the application of the theory of endomorphism rings to the abstract theory of rings: If R is an abstract ring with an identity element, we denote by a_r (by a_l) for a in R the linear transformation which maps the element x in R onto xa (onto ax); if we consider the additive group R_+ of R as an operator group with respect to the a_l, the ring of the a_r is just the full endormorphism ring and it is essentially the same as the original abstract ring R.

It would lead us too far to give a halfway complete description of the wealth of material covered in this book. It must suffice to mention some of the more salient facts.