CLASSES OF SEQUENCES OF POSITIVE NUMBERS

E. H. MOORE[1]

The results presented in the following pages engaged the attention of Professor E. H. Moore at various times, but were never written up for publication. Notes dated December 1909 indicate that the impetus came in connection with utilization of the "no last absolutely convergent series" theorem of Du Bois-Reymond (see *General analysis*, p. 48) and the paper of Landau in Nachr. Ges. Wiss. Göttingen (1907) pp. 25–27) in which Landau proves that if $\{a_n\}$ is a sequence such that $\sum |a_nb_n|$ converges for all sequences such that $\sum |b_n|^p$, with $p>1$, converges, then $\sum |a_n|^{p/(p-1)}$ converges. This latter theorem proved interesting not only because it is a sort of converse to the theorem: If $\sum |a_n|^p$ converges and if $\sum |b_n|^{p/(p-1)}$ converges then $\sum |a_nb_n|$ converges, a consequence of Hoelder's inequality, but also because the theorem which Landau actually proved was the contrapositive equivalent theorem, namely, if $\sum |a_n|^{p/(p-1)}$ diverges, then there exists a sequence such that $\sum |b_n|^p$ converges, but $\sum |a_nb_n|$ diverges. By noting the trivial identity $a_n=(a_nb_n)(1/b_n)$, the last theorem has the skeleton form: If the sequence $\{a_n\}$ belongs to a class $\mathfrak{M}_1$ (namely, the divergent $p/(p-1)$ power) then there exist two sequences $\{b_n\}$ and $\{c_n\}$ such that $a_n=b_nc_n$, and $\{b_n\}$ belongs to $\mathfrak{M}_2$ (convergent p the power) and $\{c_n\}$ to $\mathfrak{M}_3$ (divergent). Since logical questions were always of great interest to him, this instance in which the contrapositive of a given theorem has an independently interesting statement led him to speculations concerning the true nature of contrapositive proof. Among other things he stressed the idea that many so-called contrapositive proofs could be formulated to advantage as direct proofs.

From his notes, it appears that the subject matter of the first half of this paper engaged his continued attention from December 1910 to January 1911. But apparently he made no attempt to write up the results for publication until about 1918. However, only a few sections of the manuscript were completed. On several occasions he worked out a "multiplication table" (see end of Part II) and added to it from time to time. We have not always been able to connect the references which some of these tables contain with his notes, and consequently reconstruct his proofs. Instead we have used the final re-

Received by the editors July 11, 1945.

[1] Prepared for publication by H. H. Goldstine and T. H. Hildebrandt.