A NOTE ON WEAK DIFFERENTIABILITY OF PETTIS INTEGRALS

M. EVANS MUNROE

Pettis[1] raised the question whether or not separability of the range space implies almost everywhere weak differentiability of Pettis integrals. Phillips[2] has given an example which answers this question in the negative. His construction is based on a sequence of orthogonal vectors in Hilbert space. We present here a different example of the same type of function. Our basic construction is that of a function defined to the space C. Using that function as a basis, we are able to give a specific construction of such a function defined to each member of a large class of Banach spaces.

1. Metric density properties of a non-dense perfect set. Let $B \subset [0, 1]$ be a non-dense perfect set of measure one-half, and let $\overline{B}$ be its complement. $\overline{B}$ may be constructed by taking the sum of a set of open intervals classified as follows:

1 interval of length 1/4,
2 intervals each of length 1/16,
4 intervals each of length 1/64,
. . . .
2^{n-1} intervals each of length $1/2^{2n}$,
. . . .

We shall refer to the intervals of length $1/2^{2n}$ as *intervals of* $\overline{B}$ *of order* n. We shall assume that each interval of $\overline{B}$ of order n is the center portion of the space either between two intervals of $\overline{B}$ of lower order or between one such interval of $\overline{B}$ and an end point of the unit interval. These spaces we shall refer to as *gaps of order* n, and we shall denote such a gap by the symbol G_n. If $\overline{B}$ is constructed as noted above, then for each n, any two sets each of the form $G_n \cdot \overline{B}$ are congruent; hence we shall use G_n to denote a gap of order n, and we shall not find it necessary to specify which one.

The following three lemmas are now obvious.

1.1. LEMMA. $|\overline{B} \cdot G_n| = 1/2^{2n-1}$.

1.2. LEMMA. $|G_n| = 1/2^n + 1/2^{2n-1}$.

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[1] See [3, p. 303]. Numbers in brackets refer to the references cited at the end of the paper.

[2] See [4, p. 144].