# CONTINUED FRACTION EXPANSIONS FOR FUNCTIONS WITH POSITIVE REAL PARTS

H. S. WALL

1. **Introduction.** Let $K$ denote the region of the complex $z$-plane exterior to the cut along the real axis from $-1$ to $-\infty$. Let $E$ denote the class of functions $F(z)$ with the following three properties:

$$\text{(1.1)}\qquad \begin{aligned} &\text{(a)}\quad F(z) \text{ is analytic over } K;\\ &\text{(b)}\quad F(0) = 1;\\ &\text{(c)}\quad R(F(z)) > 0 \text{ over } K. \end{aligned}$$

The object of this paper is to prove that the class $E$ is coextensive with the class of functions representable in the form

$$\text{(1.2)}\qquad \cfrac{(1+z)^{1/2}}{1+\cfrac{g_1 z}{1+ir_1(1+z)^{1/2}+\cfrac{(1-g_1)g_2 z}{1+ir_2(1+z)^{1/2}+\cfrac{(1-g_2)g_3 z}{1+ir_3(1+z)^{1/2}+\cdots}}}},$$

where $0<g_p<1$, $-\infty<r_p<+\infty$, $p=1, 2, 3, \cdots$, or as a terminating continued fraction of this form, in which the last $g_p$ which appears may be equal to unity. The continued fractions converge uniformly over every bounded closed region within $K$. That branch of $(1+z)^{1/2}$ is to be taken in $K$ which equals 1 for $z=0$.

This result supplements [3].[1] In fact, the continued fraction (1.2) is actually the continued fraction (3.6) of [3]. At that time we did not recognize that the latter can be put in the form (1.2), and we proved convergence only in the neighborhood of the origin. If $r_p=0$, $p=1, 2, 3, \cdots$, the continued fraction (1.2) reduces to a familiar form first considered by E. B. Van Vleck [2], and recently by the present writer [4] in connection with totally monotone sequences. From one point of view, the result is a reformulation of a theorem of Schur [1] on bounded analytic functions.

Presented to the Society, September 17, 1945, under the title *Analytic functions with positive real parts*; received by the editors July 9, 1945.

[1] Numbers in brackets refer to the Bibliography at the end of the paper.