RECTILINEAR CONGRUENCES WHOSE DEVELOPABLES INTERSECT A SURFACE IN ITS LINES OF CURVATURE

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Introduction. It is well known that the developable surfaces of the congruence of normals to a surface intersect the surface in its lines of curvature. One may inquire if there exist congruences other than the congruence of normals to a surface the developables of which intersect the surface in its lines of curvature. It is the chief aim of this paper to give an affirmative answer to this query. The exhibition of a congruence of the required variety depends upon the solution of a partial differential equation of Laplace—a circumstance which occurs frequently in problems of differential geometry.

The notation employed here is that of Eisenhart,[1] with the exception that $\Gamma^{\alpha}_{\beta\gamma}$ will be used for the Christoffel symbol of the second kind. Greek letters will take the range 1, 2, and Latin letters the range 1, 2, 3. The convention of the tensor analysis as to summation on repeated indices will be observed.

1. **Analytical development.** Consider a surface S represented by $x^i = x^i(u^1, u^2)$ $(i = 1, 2, 3)$ referred to a rectangular cartesian system of coordinates. The functions $x^i(u^1, u^2)$, together with their partial derivatives to the second order, are understood to be continuous at any point P of the surface. A unique line λ of a congruence Λ is determined at each point P of the surface S by the direction cosines

$$\lambda^i = \lambda^i(u^1, u^2), \qquad \lambda^i\lambda^i = 1, \tag{1}$$

where the functions λ^i and their first partial derivatives are continuous at points of the surface under consideration.

The functions λ^i may be expressed in terms of the direction numbers $x^i_{,\alpha}$ $(\alpha = 1, 2)$ of the tangents to the coordinate curves on the surface through P, and the direction cosines X^i of the normal to the surface at P. Thus,

$$\lambda^i = p^{\alpha} x^i_{,\alpha} + qX^i, \tag{2}$$

where p^{α} are the contravariant components of a vector in the surface at P, q is a positive scalar function, and $x^i_{,\alpha}$ denotes covariant

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[1] Eisenhart, *Differential geometry*, Princeton University Press, 1940.