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# ON ISOMETRIES OF SQUARE SETS

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**1. Introduction.** It is not fully known under what conditions the isometry of two square, metric sets, say $E^2$ and $F^2$, implies the isometry of $E$ and $F$. Using the notion of order two self-isometries, this paper gives conditions sufficient to imply $E$ isometric to $F$ when $E^2$ and $F^2$ are finite and are metrized under any one of a fairly extensive class of functions. The basic ideas are first applied to non-square sets to yield a more general theorem which is then applied to the inverse square problem.

**2. Definitions.** A set is called metric if to every pair of its elements, $a$ and $b$, there corresponds a real, non-negative number, which is independent of the order of $a$ and $b$, zero if and only if $a$ equals $b$, and which satisfies the triangle law.

Two metric sets are isometric (written "$\equiv$") if there is a one-to-one transformation of one set on the other in which the metric number associated with any pair is the same as that associated with the transformed pair.

A non-identity mapping of a set on itself, which is an isometry, and which leaves each element of the set invariant or else interchanges it with another, is called a self-isometry of order two. Any subset on which the self-isometry is the identity is said to be left pointwise invariant.

THEOREM 1. *Assume $A\equiv B$ under a mapping $T$, where $A$ and $B$ are finite metric sets. Let $A$ and $B$ have self-isometries of order two under mappings $R$ and $S$ respectively and let $A_1$ and $B_1$ denote respectively the maximum subsets left pointwise invariant. If $A_1$ has no self-isometry of order two, and has at least as many elements as $B_1$, then $A_1\equiv B_1$ and there*

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