ON THE HAMILTON DIFFERENTIAL

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1. **Introduction.** In the absolute geometric development of vector analysis Hamilton found it necessary to formulate a definition for the differential of a point function, since division by a vector is excluded in vector analysis. It is the purpose of this note to relate a restricted form of the Hamilton differential to that of Stolz and another modified form to a differential defined by Rainich.

2. **The Hamilton differential.** The definition of Hamilton for the differential ϕ' of a point function $\phi(P)$ may be expressed by

$$\phi'(P, dP) = \operatorname*{limit}_{\lambda \to 0} \frac{\phi(P + \lambda dP) - \phi(P)}{\lambda}. \tag{2.1}$$

This definition is not entirely satisfactory. For some functions it furnishes differentials which are usually considered as nonexistent. For example, $(dxdy)^{1/2}$ is the differential of $(xy)^{1/2}$ at the origin according to definition (2.1). That such situations arise is due to the fact that the differential here defined does not necessarily possess the linearity property which will be defined later. This defect was recognized by Rainich[1] who proposed another form of the Hamilton differential which possesses this desired property. This is essential if the differential of a tensor point function is to be again a tensor.

3. **The Rainich differential.** Instead of making a single point $P + \lambda dP$ approach P as λ goes to zero Rainich makes each of two points Q_λ and P_λ approach P as λ goes to zero. His definition may be formulated as follows:

DEFINITION (3.1).

(a) $$\phi'(P, dP) = \operatorname*{limit}_{\lambda \to 0} \frac{\phi(Q_\lambda) - \phi(P_\lambda)}{\lambda},$$

which limit must exist for all modes of approach of Q_λ and P_λ to P for which

(b) $$\operatorname*{limit}_{\lambda \to 0} \frac{Q_\lambda - P_\lambda}{\lambda} = dP.$$

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[1] G. Y. Rainich, Amer. J. Math. vol. 46 (1924) p. 78.