SOLUTION OF A CLASS OF SINGULAR INTEGRAL EQUATIONS

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The following class of integral equations may be of some importance in the applications:

$$(1)\qquad g(x) = \frac{1}{\pi}\oint_{-1}^{1} f(\xi)\left\{\frac{1}{\xi - x} + \sum_{n=0}^{N} c_n(\xi - x)^{2n+1}\right\} d\xi.$$

The symbol $\oint$ indicates that the principal value of the integral is to be taken and the coefficients c_n are given constants. The special case of all $c_n = 0$ has been dealt with extensively, for instance by Glauert [1], Fuchs [2], Hamel [3], Schroeder [4] and Söhngen [5].[1] The values of the coefficients c_n might be determined by the condition that a given kernel $K(\xi - x)$, for instance $K = 1/\sinh(\xi - x)$, is approximated as nearly as possible by the kernel of equation (1).

The purpose of the present note is to derive the solution of (1) for a finite number of nonvanishing c_n. The method of solution is an extension of the method applicable when all $c_n = 0$.

Equation (1) is first transformed by the substitutions

$$(2)\qquad x = \cos\phi, \qquad \xi = \cos\theta,$$

$$(3)\qquad g(x) = G(\phi), \qquad f(\xi) = F(\theta)$$

into

$$(4)\quad G(\phi) = \frac{1}{\pi}\oint_{0}^{\pi} F(\theta)\left\{\frac{1}{\cos\theta - \cos\phi} + \sum_{n=0}^{N} c_n(\cos\theta - \cos\phi)^{2n+1}\right\} \sin\theta d\theta.$$

The function $G(\phi)$ is thought to be developed in the interval $(0, \pi)$ in the following form:

$$(5)\qquad \sin\phi\, G(\phi) = \sum_{m=1}^{\infty} B_m \sin m\phi.$$

It is then to be shown that the following representation of $F(\theta)$

$$(6)\qquad \sin\theta\, F(\theta) = \sum_{m=0}^{\infty} A_m \cos m\theta$$

permits the explicit determination of the unknown coefficients A_m in

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[1] Numbers in brackets refer to the references cited at the end of the paper.