NULL SYSTEMS IN PROJECTIVE SPACE

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If P is an (abstract) n-dimensional projective space, then we define a *polarity* in P as a correspondence p associating with every point Q in P a hyperplane Q^p and with every hyperplane h in P a point h^p in such a way that:

(i) $Q = Q^{p^2}$ for every point Q and $h = h^{p^2}$ for every hyperplane h.

(ii) The point Q is on the hyperplane h if, and only if, the hyperplane Q^p passes through the point h^p.

It is an immediate consequence of (i) that polarities are 1:1 correspondences.

We shall term p a *null-polarity* if the polarity p has the additional property that:

(iii) Every point Q is on the corresponding hyperplane Q^p, and consequently every hyperplane h passes through the corresponding point h^p.

Extending a result of Veblen and Young, R. Brauer[1] has shown that the existence of a null-polarity in P implies that the number n of dimensions of P is odd, and he has connected the null-polarities with the so-called null-systems, provided P is the n-dimensional projective space over a commutative field of coordinates. It is the object of the present note to show that this last hypothesis may be omitted; more precisely we are going to show that if the dimension of P is greater than 1, then the existence of a null polarity is equivalent to the fact that P is of odd dimension and is a projective space over a commutative field of coordinates.

If P is a projective space of dimension 1, then the hyperplanes are points too. The identity transformation on the points of the line P is therefore the null-polarity of P. For this reason we shall assume throughout the remainder of this note that P be of dimension greater than 1.

The case of a projective plane P has to be treated separately from the others, since the Theorem of Desargues need not hold true in a projective plane, though it is true for all the higher-dimensional projective spaces.

A projective plane is a system of points and lines such that any two different lines meet in one and only one point, any two different

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[1] R. Brauer, *A characterization of null systems in projective space*, Bull. Amer. Math. Soc. vol. 42 (1936) pp. 247–254.