§16) to show that intuitionistic number theory admits, besides the extension which gives classical number theory, also an intuitionistic extension. Both extensions are simply consistent if the unextended system is, and the two extensions are contradictory to each other. The work involves formalization of the theory of certain primitive recursive predicates $R_k(e, x_1, \cdots, x_k, y)$ which in a formal intuitionistic system afford a representation of the theory of general and partial recursive functions and predicates. The present results combine with reasoning of Kleene (Abstract 48-1-85) to establish the independence of certain formulas of the intuitionistic predicate calculus, in particular of the formula $\neg\neg(x)(A(x) \vee \neg A(x))$. (Received August 8, 1945.)

Statistics and Probability

241. Reinhold Baer: *Sampling from a changing population.*

The stochastic limits of certain functions of random samples are determined where the samples are taken from different distributions belonging to a continuous family of distributions. (Received August 22, 1945.)

242. J. L. Doob: *Markoff chains—denumerable case.*

Let $p_{ij}(t)$, $i, j=1, 2, \cdots, 0 \leqq t < \infty$, be the transition probability functions of a Markoff process. Let $x(t)$ be the (integral) value assumed by the probability system at time t. Necessary and sufficient conditions are found that the $p_{ij}(t)$ satisfy the systems of first order differential equations (*) $p'_{ik}(t) = -q_i p_{ik}(t) + \sum_{j \neq i} q_{ij} p_{jk}(t)$, $p'_{ik}(t) = -p_{ik}(t) q_k + \sum_{j \neq i} p_{ij}(t) q_{jk}$, where $q_i = -p'_{ii}(0)$, $q_{ik} = p'_{ik}(0)$ $(i \neq k)$. A detailed analysis is made of the processes for which the discontinuities of $x(t)$ are well ordered. It is shown that if q_i, q_{ik} are specified arbitrarily except that $q_{ik} \geqq 0$, $q_i = \sum_j q_{ij}$, there is always a corresponding set of functions $\{p_{ij}(t)\}$ determining a Markoff process, but that in general there will be infinitely many such sets of functions, and even infinitely many satisfying (*), such that the discontinuities of $x(t)$ are well ordered. The initial conditions $p_{ij}(0) = \delta_{ij}$ are thus insufficient to determine uniquely the solutions to (*). (Received September 22, 1945.)

243. Mark Kac: *On the average of a certain Wiener functional.*

Let $x(t)$ be an element of the Wiener space. It is shown that the average of the functional $\exp\left(-z\int_0^1 |x(t)|\,dt\right)$ $(z>0)$ is given by the formula $\sum_1^\infty \kappa_j \exp\left(-(\delta_j/2)z^{2/3}\right)$, where $\delta_1, \delta_2, \cdots$ are positive zeros of the derivative of $P(y) = (2y)^{1/2}\{J_{-1/3}((2^{3/2}/3)y^{3/2}) + J_{1/3}((2^{3/2}/3)y^{3/2})\}$ and $\kappa_j = (1 + \int_0^{\delta_j} P(y)dy)/\delta_j P(\delta_j)$. A related limit theorem in calculus of probability is discussed. In the course of the proof the following seemingly new result was also obtained: If r_j is the jth positive root of $J_\nu(x)$ $(\nu \geqq 0)$ then $r_j^2 > \nu^2 + (2\pi j)^{2/3}\nu^{4/3}$. For j fixed the estimate is weaker than a known asymptotic formula. The value of the estimate is due to the fact that j can depend on ν. (Received August 2, 1945.)

244. Isaac Opatowski: *Calculation of Markoff chains by incomplete gamma and beta functions and by Charlier polynomials.*

Several types of stochastic processes consisting of successive transitions between n states $\{i\}_1^n$ are considered (cf. Bull. Amer. Math. Soc. vol. 51 (1945) p. 665). Call $P_{r,s}(t)$ the probability of being a system at the time t in the state s if it is at $t=0$ in the state r. Let the only transitions possible during dt be $(i-1 \rightarrow i)(i+1 \rightarrow i)$ and the