BOOK REVIEWS

The problem of moments. By J. A. Shohat and J. D. Tamarkin. (Mathematical Surveys, vol. 1.) New York, American Mathematical Society, 1943. 140 pp. $2.25.

This book is the first in a new series sponsored by the American Mathematical Society. Each number of the series is evidently designed to collect all important existing material on a single mathematical discipline and to expound it in a readable and understandable manner. If so, then the volume under review succeeds admirably for that branch of mathematics which has recently grown up about a problem posed and solved by T. J. Stieltjes and named by him "the moment problem." The authors have set a high standard of excellence in presentation and choice of material, which may well establish the tone for the series.

The moment problem may be stated as follows: Given a sequence of real numbers $\mu_0, \mu_1, \cdots$; determine a nondecreasing function $\psi(t)$ such that

$$\mu_n = \int_a^b t^n d\psi(t), \qquad n = 0, 1, 2, \cdots . \tag{1}$$

If $a=0$, $b=\infty$, the problem is precisely as Stieltjes set it and is known as the Stieltjes problem; it is known by the names of F. Hausdorff or H. Hamburger according as $a=0$, $b=1$ or $a=-\infty$, $b=+\infty$. Of course the first two are special cases of the last, but it is profitable to study them separately. The case in which $\psi(t)$ is to be the Riemann integral of a positive function was studied by earlier authors, notably E. Heine and P. Tchebycheff. In spite of this it seems proper that the name of Stieltjes should be attached to the problem, for the generalized integral which he introduced, now known as the Riemann-Stieltjes integral, was indeed a very happy idea for the development of the problem. Without it finite linear combinations and integrals would have to be studied separately, and nothing like the existing elegance in the theory could possibly be attained.

The reason for the term "moment" becomes evident if one interprets $\psi(t)$ as defining a distribution of mass along the interval $a \leqq t \leqq b$. Then μ_0 is the total mass; μ_1 is the statical moment which measures the tendency of the segment to turn about the origin if held horizontally; μ_2 is the moment of inertia about the origin. Stieltjes defines μ_n as the nth moment, and his fundamental question