

ing conics of a plane curve when referred to a general reference frame. The method employed makes possible a unification of types of osculants which are seemingly quite diverse. (Received July 19, 1945.)

181. H. P. Pettit: *On the generation of certain algebraic surfaces.*

A surface of order  $2mn$  is the locus of the curve of intersection of two cones in which the intersections of a plane of a pencil with a base surface of order  $m$  and a base surface of order  $n$  are projected from two fixed points. These fixed or base points are  $mn$ -fold points on the generated surface, the tangent cones consisting, respectively, of  $m$  cones of order  $n$  and  $n$  cones of order  $m$ . The surface contains a plane  $n$ -ic as an  $m$ -fold curve and a plane  $m$ -ic as an  $n$ -fold curve. For  $m=n=1$  the method is the ordinary projective generation of the ruled quadric. For a particular choice of the base points relative to the base surfaces certain degeneracies take place in the generated surface. In a plane through the base points, the process produces the method of generating plane curves which was discussed by the author in *The projective description of some higher plane curves*, Tôhoku Math. J. vol. 27 (1926). (Received May 26, 1945.)

LOGIC AND FOUNDATIONS

182. Garrett Birkhoff: *Universal algebra.*

An unpublished result of Bruce Crabtree is extended to show that, if  $A$  is any algebra with finitary operations, and  $G$  is any subset and  $S$  any subalgebra of  $A$ , then there is a maximal subalgebra  $T$  satisfying  $G \cap T \leq S$ . If the lattice of subalgebras of  $A$  is distributive, then it must satisfy  $X \cap \bigcup Y_\alpha = \bigcup (X \cap Y_\alpha)$ . Hence not every complete lattice is the lattice of all subalgebras of a suitable universal algebra. (Received July 5, 1945.)

183. R. M. Robinson: *Finite sequences of classes.*

This note discusses the definition of a finite sequence of classes, in an axiomatic set theory in which "sets" and "classes" are distinguished, only sets being allowable as elements. (Received July 23, 1945.)

STATISTICS AND PROBABILITY

184. Isaac Opatowski: *Direct and reverse transitions in Markoff chains.*

The author considers stochastic processes consisting of successive transitions between  $n+1$  states  $\{i\}_0^n$  according to the law  $dP_i/dt = k_i P_{i-1} - k_{i+1} P_i + g_i P_{i+1} - g_{i-1} P_i$ ,  $P_0(0) = 1$ ,  $P_i(0) = 0$  for  $i \geq 1$ , where  $P_i(t)$  is the probability that the system be in the state  $i$  at the time  $t$  if it is at the time  $t=0$  in the state 0. The constants  $k_i$  and  $g_i$  represent respectively the "intensities" of the direct and reverse transitions ( $i-1 \rightarrow i$ ) ( $i+1 \rightarrow i$ ).  $k_1 > 0$ ,  $g_1 \geq 0$  for any  $i$ , except  $k_0 = g_n = 0$ . It is shown that if  $k_{n+1} = g_{n-1} = 0$ , and consequently  $\sum_{i=0}^{i=n} P_i = 1$ , the process is equivalent, as far as the probability  $P_n(t)$  is concerned, to a new process of the same type, between the same number of states consisting, however, of direct transitions only with intensities  $\{\bar{k}_i\}_1^n$ . The main part of the proof consists in showing that  $\{-\bar{k}_i\}_1^n$ , which are the poles of the Laplace transform of  $dP_n/dt$ , are all real and negative. They are roots of the determinant  $\|a_{i,j}\|_n$  where  $a_{i,i} = x + k_{i+1} + g_{i-1}$ ;  $a_{i,i+1} = g_i$ ,  $a_{i,i-1} = k_i$  with all the other  $a_{i,j}$ 's zero. The