151. A. N. Milgram: *Cyclotomically saturated polynomials and tri-operational algebra.*

Let $f(x)$ be a polynomial whose coefficients are integers mod p where p is a prime number. Call $f(x)$ cyclotomically saturated if it has the property that for each irreducible polynomial $\phi(x)$, if $[\phi(x)]^{\nu} | f(x)$, then also $(x^{p^n}-x)^{\nu} | f(x)$ where n is the degree of $\phi(x)$. In tri-operational algebra (cf. Reports of a Mathematical Colloquium, nos. 5–6, p. 5) Menger raised the question: What polynomials with coefficients over the integers mod p have the property $f(x) | f(g(x))$ for each polynomial $g(x)$? The answer is: $f(x) | f(g(x))$ for each $g(x)$ if and only if $f(x)$ is cyclotomically saturated. (Received June 23, 1945.)

152. J. M. H. Olmsted: *Transfinite rationals.*

As suggested by the treatment of ratios by Eudoxus, two cardinal number pairs, (a, b) and (c, d), are defined to be equivalent if and only if for every pair of cardinal numbers, m and n, ma and nb have the same order relation as mc and nd. Addition, multiplication, division, and ordering are defined among the equivalence classes of cardinal number pairs, the resulting system being a lattice with familiar algebraic laws (for example, multiplication is distributive over addition, joins, and meets). This system is an extension of both the positive rational numbers and the cardinal numbers. Furthermore, it is the smallest extension subject to certain conditions. (Received June 4, 1945.)

153. Gordon Pall: *Hermitian quadratic forms in a quasi-field.*

Let F be a quasi-field, B_1 and B_2 nonsingular hermitian matrices of order $n-1$ in F, and let a be a nonzero scalar. Let there be given a transformation of $\bar{x}_0 a x_0 + \bar{x}' B_1 x$ into $\bar{x}_0 a x_0 + \bar{x}' B_2 x$. Then explicit transformations are constructed which replace B_1 by B_2. This is an extension of a similar result due to Witt for fields. (Received July 23, 1945.)

ANALYSIS

154. E. F. Beckenbach: *On a characteristic property of linear functions.*

Let there be given a class of real functions $\{f(x)\}$, defined and continuous in a closed and bounded interval, such that there is a unique member of the family which, at arbitrary distinct x_1, x_2 in the interval, takes on arbitrary values y_1, y_2 respectively. The class of linear functions is an example. It is shown that a real function $g(x)$, defined and continuous in the interval, is a member of $\{f(x)\}$ if and only if for each x_0 interior to the interval there exists an $h_0 = h_0(x_0)$ with $x_0 \pm h_0$ in the interval such that the member of $\{f(x)\}$ coinciding with $g(x)$ at $x_0 \pm h_0$ coincides with $g(x)$ also at x_0. (Received June 21, 1945.)

155. Stefan Bergman: *Pseudo harmonic vectors and their properties.*

The author applies the operator $\mathfrak{P}(f, \mathfrak{Q}, \mathfrak{T})$ introduced in Bull. Amer. Math. Soc. (vol. 49 (1943) p. 164) to complex functions $f = s^{(1)}(x, y) + is^{(2)}(x, y)$, for which $s_y^{(1)} = s_x^{(2)}$, $s_x^{(1)} = -s_y^{(2)} l(x)$ holds. Here $s_x^{(1)} = (\partial s^{(1)}/\partial x), \cdots$ and $l(x)$ is an analytic function of a real variable x. $\mathfrak{P}(s^{(1)} + is^{(2)}, \mathfrak{Q}, \mathfrak{T})$ yields a three-dimensional vector $\mathfrak{S}(X, Y, Z) = \mathfrak{S}^{(1)} + i\mathfrak{S}^{(2)} = \sum_{k=1}^{3}(S^{(k1)} + iS^{(k2)})\mathfrak{i}_k$ for which curl $\mathfrak{S}^{(1)} = 0$, $S_X^{(11)}$