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ILLINOIS INSTITUTE OF TECHNOLOGY AND
BROWN UNIVERSITY

INTEGRAL DISTANCES

NORMAN H. ANNING AND PAUL ERDÖS

In the present note we are going to prove the following result:

For any n we can find n points in the plane not all on a line such that their distances are all integral, but it is impossible to find infinitely many points with integral distances (*not all on a line*).[1]

PROOF. Consider the circle of diameter 1, $x^2+y^2=1/4$. Let $p_1, p_2, \cdots$ be the sequence of primes of the form $4k+1$. It is well known that

$$p_i^2 = a_i^2 + b_i^2, \qquad a_i \neq 0, \qquad b_i \neq 0,$$

is solvable. Consider the point (on the circle $x^2+y^2=1/4$) whose distance from $(-1/2, 0)$ is b_i/p_i. Denote this point by (x_i, y_i). Consider the sequence of points $(-1/2, 0)$, $(1/2, 0)$, (x_i, y_i), $i=1, 2, \cdots$. We shall show that any two distances are rational. Suppose this has been shown for all $i<j$. We then prove that the distance from (x_j, y_j) to (x_i, y_i) is rational. Consider the 4 concyclic points $(-1/2, 0)$, $(1/2, 0)$, (x_i, y_i), (x_j, y_j); 5 distances are clearly rational, and then by Ptolemy's theorem the distance from (x_i, y_i) to (x_j, y_j) is also rational. This completes the proof. Thus of course by enlarging the radius of the circle we can obtain n points with integral distances.

It is very likely that these points are dense in the circle $x^2+y^2=1/4$, but this we can not prove. It is easy to obtain a set which is dense on $x^2+y^2=1/4$ such that all the distances are rational. Consider the

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[1] Anning gave 24 points on a circle with integral distances. Amer. Math. Monthly vol. 22 (1915) p. 321. Recently several authors considered this question in the Mathematical Gazette.