2. C. J. Everett, *Vector spaces over rings*, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 312–316.
3. B. L. van der Waerden, *Moderne Algebra*, vol. 2, 1st ed., Berlin, 1931.

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ON A CONSTRUCTION FOR DIVISION ALGEBRAS OF ORDER 16

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It is not known whether there exist division algebras of order 16 (or greater) over the real number field $\mathfrak{R}$. In discussing the implications of this question in algebra and topology, A. A. Albert told the author that the well known Cayley-Dickson process[1] does not yield a division algebra of order 16 over $\mathfrak{R}$ and suggested a modification of that process which might. It is the purpose of this note to show that, while Albert's construction can in no instance yield such an algebra over $\mathfrak{R}$, it does yield division algebras of order 16 over other fields, in particular the rational number field R.

Initially consider an arbitrary field F. Let C be a Cayley-Dickson division algebra of order 8 over F. Define[2] an algebra of order 16 over F with elements $c=a+vb$, $z=x+vy$ (a, b, x, y in C) and with multiplication given by

$$cz = (a + vb)(x + vy) = (ax + g\cdot ybS) + v(aS\cdot y + xb) \tag{1}$$

where S is the involution $x\rightleftarrows xS=t(x)-x$ of C and g is some fixed element of C. The Cayley-Dickson process is of course the instance $g=\gamma$ in F.

For A to be a division algebra over F the right multiplication[1] R_z must be nonsingular for all $z\neq 0$ in A. Now

$$R_z = \begin{pmatrix} R_x & SR_y \\ SL_yL_g & L_x \end{pmatrix}$$

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[1] See [1] and [2] for background and notations. Numbers in brackets refer to the references cited at the end of the paper.

[2] We should remark that this modification of the Cayley-Dickson process does yield non-alternative division algebras of orders 4 and 8 over $\mathfrak{R}$ when applied to the algebras of complex numbers and real quaternions instead of to C. See R. H. Bruck, *Some results in the theory of linear non-associative algebras*, Trans. Amer. Math. Soc. vol. 56 (1944) pp. 141–199, Theorem 16C, Corollary 1, for a generalization.