

There are indications that these coefficients can be used to extend the solution of a second order partial differential equation of the form $u_{xx} = u_t + \psi(u, x, t)$, provided that it is known at a rectangular array of points in the x, t -plane and at two other points in the next row of values of t . (Received April 30, 1945.)

GEOMETRY

129. P. O. Bell: *Metric properties of a class of quadratic differential forms.*

In the present paper a new invariant quadratic differential form Ω is geometrically defined for a general pair of surfaces S, S' whose corresponding points x, x' determine the metric normal to S at x . The ratio of the form Ω to the first fundamental form ds^2 of S , in which Ω and ds^2 are defined for a common arc element of S at x , is found to be independent of the direction of the element if and only if the surface S' is the locus of the center of mean curvature of S ; the ratio thus determined is the Gaussian curvature K of S at x . It is proved that the form Ω for an arbitrary arc element is identical with the form Kds^2 for either "conjugate" element if and only if the surface S' is the plane net at infinity. The principal directions at x of the tensor whose components are the coefficients of the form Ω are the classical principal directions of S at x for an arbitrary choice of S' . Finally, the net of lines of mean-curvature of S and the mean-conjugate net of S are characterized as integral nets of equations of the form $\Omega = 0$, for suitable selections of S' . The author employs dual systems of linear equations of the first order with the use of a tensor notation. (Received May 19, 1945.)

130. L. M. Blumenthal: *Characterization of ϕ -spherical subsets and pseudo sets.*

The class of ϕ -spherical spaces is defined by four metric postulates involving an arbitrary function ϕ . The class contains, for example, those spaces derived from the surface of the ordinary sphere in euclidean $(n+1)$ -space by making it metric with respect to geodesic (shorter arc) distance and with respect to euclidean (chord) distance. The paper develops the metric geometry of this class of spaces and obtains the metric characterizations of the subsets and pseudo sets. (The paper is to appear (in Spanish) in *Revista de Universidad Nacional de Tucumán*, Ser. A. vol. 5 under the title *La caracterización métrica de espacios ϕ -esféricos.*) (Received May 18, 1945.)

131. L. M. Blumenthal: *Metric study of elliptic spaces.*

Among the properties of elliptic spaces which make inapplicable the procedures usually employed in a metric study are (1) the "unusual" character of the locus of points equidistant from two points, (2) the lack of free movability in the large, (3) the necessity of distinguishing between "contained in" and "congruently contained in," (4) the notions of dependence and independence of subsets, if defined in the ordinary way, are not metrically invariant, and (5) the abnormal behavior with respect to equilateral subsets. The writer presents a new approach in which the metrically invariant notions of *relative independence* (*dependence*) and *class independence* (*dependence*) play fundamental roles. In terms of these notions the elliptic line is metrically defined. The extension to plane, and so on, is then possible in conventional manner. Necessary and sufficient conditions (of a *quasi-metric* nature) in order that congruent subsets of an elliptic space be superposable are obtained. (Received May 19, 1945.)

132. S. S. Chern: *Characteristic classes of Hermitian manifolds. I.*