A FIXED-POINT THEOREM

A. D. WALLACE

Introduction. The purpose of this note is to sharpen a recent result of G. E. Schweigert [4].[1] It will be shown that the condition of semi local-connectedness may be dropped. However, if this is strengthened to local-connectedness, then the conclusion asserts the existence of a fixed point. Further, though perhaps of less interest, it is shown that separability is not necessary.

In a second section we give a somewhat more abstract version which is valid for certain partially ordered topological spaces. So far as is known this is the first result of this type to appear in the literature.

1. **Schweigert's theorem.** It is assumed that S is a compact (that is, bicompact) Hausdorff space, connected and nondegenerate. Moreover T is a topological transformation of S onto itself, $TS=S$.

THEOREM. *If e is an end point of S fixed under T, then there exists a continuum $K\subset S-e$ invariant under T. Further, no point of S separates any pair of points of K in S.*

PROOF. Since e is an end point it is readily seen that we can find a point y such that

$$(1)\quad S = A + B, \qquad e \in A, \qquad y + T^{-1}y \subset B, \qquad A\cdot B = z \in S,$$

with A and B nondegenerate proper subcontinua of S. We infer that

$$(2)\quad S = TA + TB, \qquad e \in TA, \qquad y \in TB, \qquad TA\cdot TB = Tz,$$

so that $S=(A+TA)+B\cdot TB$. Clearly $A+TA$ is a continuum and hence [5] so is

$$(A + TA)\cdot B\cdot TB = z\cdot TB + Tz\cdot B.$$

We then have (supposing that $z\neq Tz$) either (a) $z\in TB$ and $Tz\in S-B$ or (b) $Tz\in B$ and $z\in S-TB$. In the first case $T^{-1}z\in B$ and we are able to apply the same argument that we use in the second case if we use T^{-1} in place of T. We therefore assume (b) and readily verify that $A\subset TA$, $TB\subset B$.

Using induction it follows that we may write

Received by the editors December 29, 1944.

[1] Numbers in brackets refer to the Bibliography at the end of the paper.