NOTE ON CONVEX CURVES ON THE HYPERBOLIC PLANE

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1. **Introduction.** In a previous note [5][1] we have obtained some properties referring to convex curves on the sphere. Following an analogous way our purpose is now to obtain the same properties for convex curves on a surface of constant negative curvature $K=-1$, or, what is equivalent, for convex curves on the hyperbolic plane.

In §§6 and 7 we consider the curves of constant breadth, for which we obtain the formula (7.3) which relates the length L and area F with the breadth α.

For the curves which are not of constant breadth the formula (4.5), which contains (7.3) as a particular case, holds. But (4.5) is true only if we suppose that the curve has in all its points geodesic curvature κ_g grêater than one.

2. **Definitions.** A closed curve C on a surface of constant negative curvature $K=-1$ is said to be convex when it cannot be cut by any geodesic in more than two points, except that a complete arc of geodesic may belong to the curve. Any closed convex curve C has a finite length L and bounds a finite area F. In the following, unless otherwise specified, we shall suppose that C is composed of a finite number of arcs each with continuous geodesic curvature κ_g.

Let ω_i be the exterior angles which these arcs form at the vertices of C. Then we have the Gauss-Bonnet formula [3, p. 191],

$$\int_C \kappa_g ds + \sum \omega_i = 2\pi + F. \tag{2.1}$$

If a point O on C is taken as origin, any point A of C can be determined by the length of the arc $OA=s$ or by the angle τ defined by

$$\tau = \int_0^s \kappa_g ds + \sum_s \omega_i \tag{2.2}$$

where $\sum_s \omega_i$ is extended over all the vertices of C contained in the arc OA.

Any geodesic with only one common point or with a complete arc in common with C is called a "geodesic of support" of C. In each

Received by the editors November 18, 1944.

[1] Numbers in brackets refer to the references cited at the end of the paper.