

tive ring without nilpotent elements is a field. The purpose of the present paper is to make some further characterizations of subdirectly irreducible commutative rings. Let  $R$  be a commutative ring, not all elements of  $R$  being divisors of zero, and denote by  $D$  the set of all elements of  $R$  which are divisors of zero. Then  $R$  is subdirectly irreducible if, and only if, it has the following four properties: (i) the set of all elements  $x$  of  $R$  such that  $Dx=0$  is a principal ideal  $J=(j) \neq 0$ , (ii) the set of all elements  $y$  of  $R$  such that  $Jy=0$  is precisely  $D$ , (iii)  $R/D$  is a field, (iv) if  $d_1$  is any element of  $D$ , not in  $J$ , there exists an element  $d_2$  of  $D$ , not in  $J$ , such that  $d_1d_2=j$ . Some additional related results are also obtained. (Received February 20, 1945.)

81. Kathryn A. Morgan: *Representation of a positive binary form by a positive quaternary form.*

The conditions for representing a positive binary form as a sum of squares of linear forms were discussed by Mordell (Quart. J. Math. Oxford Ser. vol. 1 (1930) pp. 276-288) and Chao Ko (Quart. J. Math. Oxford Ser. vol. 8 (1937) pp. 81-98). This paper presents a method for finding the representation of a positive primitive binary form by a positive quaternary form and especially the number of representations of a primitive binary form as the sum of four squares. (Received March 3, 1945.)

82. F. E. Satterthwaite: *Error control in matrix calculation. II.*

The arithmetic calculation of the inverse of a matrix or of the solution of a set of simultaneous equations is often complicated by a serious magnification of rounding errors. The proposed method is as follows: (1) Each equation (or line) of the Doolittle solution is expressed approximately as an exact linear function of the original equations. (2) The discrepancy between the approximation and the ideal is adjusted by the same type of process as is used to adjust the original equations in a standard Doolittle method. (3) If the approximation is close enough, the coefficients in the adjustment will be small enough so that they can not cause any significant carrying forward of errors. A second approximation is sometimes necessary to satisfy this condition. The advantages of this method are: (1) It works for any matrix or set of equations. (2) It does not require an advance approximate solution. (3) Any number of decimal places may be carried with complete assurance that errors are never greater than one or two in the last decimal place. (4) Each step is self-checking. (5) The method is ideally suited for use with modern high speed calculating machines. (6) The routine can be easily taught to the average clerk. (Received February 5, 1945.)

83. J. E. Wilkins: *A generalization of the Euler  $\phi$ -function.*

One defines the function  $\phi_n(k)$  so that exactly  $\phi_n(k)$  of the  $k$  arithmetic progressions  $mk+l$  ( $m=0, 1, \dots; l=0, 1, \dots, k-1$ ) contain infinitely many numbers not divisible by an  $n$ th power greater than 1. It is shown that  $\phi_n(k)$  is that multiplicative function for which  $\phi_n(p^r) = p^r$  if  $r < n$  and  $p$  is prime and  $\phi_n(p^r) = p^r - p^{r-n}$  if  $r \geq n$  and  $p$  is prime. Thus  $\phi_1(k)$  is the Euler  $\phi$ -function. It is also shown that the function  $\phi_n(k)/k$  is uniformly almost periodic in the sense of Bohr for  $n \geq 2$ , and its asymptotic mean value is  $1/\zeta(2n)$ . For  $n=2$ , these results are due to Haviland, whose discussion, however, is not free from errors. (Received March 20, 1945.)

#### ANALYSIS

84. R. P. Agnew: *Spans of translations of peak functions.*

A peak function  $F(x)$  is defined, in terms of positive constants  $a$  and  $b$ , by