# NOTE ON INTERPOLATION FOR A FUNCTION OF SEVERAL VARIABLES

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The simplest interpolation formula for a function of $\omega$ variables $x, y, \cdots, z$ is the multiple Gregory-Newton formula, which approximates the function by a polynomial in $p, q, \cdots, r$ of total degree $n$, namely,

$$\begin{aligned} & f(x + ph_1, y + qh_2, \cdots, z + rh_\omega) \\ (1) \qquad & = \sum_{i+j+\cdots+k=0}^{n} \binom{p}{i}\binom{q}{j}\cdots\binom{r}{k}\Delta_{x^iy^j\cdots z^k}^{i+j+\cdots+k} f(x, y, \cdots, z), \end{aligned}$$

where $x, y, \cdots, z$ denote the independent variables, $h_m$ denotes the tabular intervals,

$$\binom{p}{i} \text{ denotes } \frac{p(p-1)\cdots(p-i+1)}{i!}, \text{ with } \binom{p}{0} = 1,$$

and $\Delta_{x^iy^j\cdots z^k}^{i+j+\cdots+k} f(x, y, \cdots, z)$ denotes the mixed partial advancing difference of $f(x, y, \cdots, z)$, of order $i$ with respect to $x$, $j$ with respect to $y$, and so on. The summation is for all sets of values of $i, j, \cdots, k$ such that $i+j+\cdots+k$ goes from 0 to $n$. Using the notation $f_{s,t,\cdots,u}$ to denote $f(x+sh_1, y+th_2, \cdots, z+uh_\omega)$, it is apparent that the multiple Gregory-Newton formula involves all values $f_{s,t,\cdots,u}$ such that $s+t+\cdots+u=0, 1, 2, \cdots, n$. Thus for the case of 2 dimensions the arguments are the $(n+1)(n+2)/2$ points forming a right triangle, vertex at $(x, y)$, and for 3 dimensions the arguments are the $(n+1)(n+2)(n+3)/6$ points forming a solid tetrahedron, vertex at $(x, y, z)$.

The purpose of the present note is to show that when (1) is expressed in the simpler form

$$(2) \quad f(x + ph_1, y + qh_2, \cdots, z + rh_\omega) = \sum_{s+t+\cdots+u=0}^{n} C_{s,t,\cdots,u} f_{s,t,\cdots,u},$$

then we have

$$(3) \quad C_{s,t,\cdots,u} = \binom{n-p-q-\cdots-r}{n-s-t-\cdots-u}\binom{p}{s}\binom{q}{t}\cdots\binom{r}{u}.$$

Thus (1) can be employed without the labor of finding all the mixed

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