ON THE BLOCH-LANDAU CONSTANT FOR SCHLICHT FUNCTIONS

RUTH E. GOODMAN

The definition of the Bloch-Landau constant $\mathfrak{A}$ is based upon the following theorem due to Bloch:[1]

THEOREM 1. *There is an absolute positive constant P with the following property: Let $f(x)$ be regular for $|x|<1$, $|f'(0)|=1$. Then the map of $|x|<1$ under $f(x)$ contains in a single sheet an open circle of radius P.*

In addition to the original Bloch constant $\mathfrak{B}$, the least upper bound of the P satisfying the above theorem, Landau[2] has defined two other constants, $\mathfrak{L}$ and $\mathfrak{A}$, in connection with this theorem. $\mathfrak{L}$ is the least upper bound of the P of Theorem 1 if the requirement that there

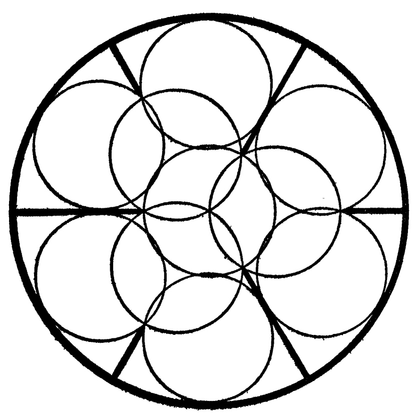

FIG. 1

be a circle of radius P contained in a single sheet is replaced by the requirement that there be a circle of radius P each point of which is an interior point of some sheet of the map. $\mathfrak{A}$ is the least upper bound of the P of Theorem 1 if it is required that the function $f(x)$ be schlicht for $|x|<1$. It is immediately apparent that $\mathfrak{B}\leqq\mathfrak{L}\leqq\mathfrak{A}$.

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[1] Bloch, *Les théorèmes de M. Valiron sur les fonctions entières, et la théorie de l'uniformisation*, C. R. Acad. Sci. Paris vol. 178 (1924) pp. 2051–2052.

[2] E. Landau, *Über die Blochsche Konstante und zwei verwandte Weltkonstanten*, Math. Zeit. vol. 30 (1929) pp. 608–634. This paper includes a proof of Theorem 1.