## BOOK REVIEWS

*Fourier series.* By G. H. Hardy and W. W. Rogosinski. (Cambridge Tracts in Mathematics and Mathematical Physics, no. 38.) New York, Macmillan; Cambridge University Press, 1944. 8+100 pp. $1.75.

The object of the authors, as stated in their preface, was to write a book on Fourier series "in a modern spirit, concise enough to be included in the (Cambridge) series" of tracts, yet complete enough to serve as an introduction to Zygmund's *Trigonometrical series.* How well their object has been achieved is readily seen on examining the wealth of material covered in some ninety pages. At the outset they define what is expected of the reader and what aspects of the subject they propose to omit. The reader is expected to have a knowledge of Lebesgue integral theory as treated in Chapters X–XII of Titchmarsh's *Theory of functions*; and while it is recognized "unofficially" that the reader will undoubtedly have some knowledge of the theory of trigonometrical series, officially the tract is self-contained.

Certain topics are frankly omitted: for example, the inequalities of Young and Hausdorff, M. Riesz's theorem on conjugate series, theorems on Cesàro summability of general order, and uniqueness theorems for summable series. Nor is Denjoy's theory of general trigonometrical series considered.

Chapter I, entitled *Generalities* lays the groundwork for all subsequent developments. At the outset trigonometrical series and trigonometrical Fourier series are defined, and at the same time their relation to harmonic and analytic functions is brought out. The emphasis on this relationship is reinforced at various points throughout the tract by the application of Fourier series theorems to analytic function theory. The essential results of Lebesgue integral theory and the theory of the spaces $L^p$ which are needed for the sequel are stated succinctly.

Chapter II is directed toward the classical $L^2$ theory of Fourier series. First the situation for general normal orthogonal systems is considered. Here the Riesz-Fischer theorem, the Parseval theorem, and the relation between closure and completeness find their natural setting. From this point on, attention is focussed on trigonometrical series. The completeness of the trigonometrical system $(T)$: $1/2$, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, $\cdots$ in $L(0, 2\pi)$ is demonstrated; and the completeness of $(x^n)$ for $n=N, N+1, \cdots$ in $L(a, b)$ and the Weierstrass approximation theorems appear as corollaries.