BI-ISOTHERMAL SYSTEMS

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1. Introduction. A pair of functions of two complex variables with nonvanishing jacobian induces a correspondence between the points of a real (or complex) four-dimensional euclidean space R_4. The infinite group G of all such correspondences is not the conformal group of R_4, which is merely the inversive group of fifteen parameters. In 1908, the senior author termed G the *pseudo-conformal group*. This is now standard terminology. (See papers by Cartan, Carathéodory, and S. Bergmann.) It will be recalled that Poincaré, in his fundamental Palermo memoir of 1907, tentatively called G the regular group.

In 1908, Kasner showed that the pseudo-conformal group G is characterized by the preservation of the pseudo-angle between any curve and a hypersurface at their common point of intersection.[1] This is a proper generalization of the well known result that the group of functions of a single complex variable is identical with the (direct) conformal group of the plane.

In R_4, there is a class of surfaces which is transformed into itself under the infinite pseudo-conformal group G such that the induced correspondence between any corresponding pair of such surfaces is (direct) conformal. Any such surface is said to be a *conformal surface.* The term *analytic surface* is also in use.

Upon projecting orthogonally (by means of absolutely perpendicular planes) a conformal surface upon a pair of selected coordinate planes, the three induced correspondences (surface on each of the two planes, and plane on plane) are each conformal. Thus any conformal surface may be defined by means of a conformality between the selected pair of coordinate planes.

We shall say that a *bi-isothermal system of* ∞^3 *curves* in R_4 is any system which is pseudo-conformally equivalent to a parallel pencil of ∞^3 straight lines in R_4. Any such system consists of ∞^2 isothermal families of ∞^1 curves, each such family lying upon a conformal surface.

We define a *bi-isothermal system of* ∞^1 *hypersurfaces* in R_4 as any system which is pseudo-conformally equivalent to a parallel pencil of ∞^1 hyperplanes in R_4. Any bi-isothermal system of hypersurfaces may be defined by placing a biharmonic function equal to an arbi-

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[1] Kasner, *Conformality in connection with functions of two complex variables,* Trans. Amer. Math. Soc. vol. 48 (1940) pp. 50–62.