THE PSEUDO-ANGLE IN SPACE OF $2n$ DIMENSIONS

JOHN DECICCO

1. Introduction. The theory of functions of a single complex variable is essentially identical with the conformal geometry of the real (or complex) plane. However, this is not the case in the theory of functions of *two or more* complex variables. Any set of $n \geqq 2$ functions of n complex variables with nonvanishing jacobian induces a correspondence between the points of a real (or complex) $2n$-dimensional euclidean space R_{2n}. The infinite group G of all such correspondences is obviously *not* the conformal group of R_{2n}, which is merely the inversive group of $(n+1)(2n+1)$ parameters.[1] Poincaré in his fundamental paper in Palermo Rendiconti (1907) has called G the group of *regular* transformations. However, in an abstract presented before the American Mathematical Society, 1908, Kasner found it more appropriate to term it the *pseudo-conformal group* G. This name is now standard.

In his work of 1908, which he later published in full in 1940, Kasner investigated the possibility of characterizing the pseudo-conformal group G of four dimensions (the case $n=2$ complex variables) in a purely geometric way.[2] His principal result is as follows: *A transformation of R_4 is pseudo-conformal if and only if it preserves the pseudo-angle between any curve and a three-dimensional hypersurface at their point of intersection.* This theorem demonstrates how the pseudo-angle may be used to characterize G within the group of arbitrary point transformations of R_4.

We shall show in this paper how Kasner's pseudo-angle theorem can be carried over to $2n$ dimensions almost without any change. The pseudo-angle is important also because all other differential invariants of the first order under the pseudo-conformal group are really combinations of this pseudo-angle.[3]

2. The minimal coordinates. Let $(x_1, x_2, \cdots, x_n; y_1, y_2, \cdots, y_n)$

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[1] The conformal group of a euclidean space R_m of any dimension $m>2$, odd or even, is the inversive group of $(m+1)(m+2)/2$ parameters (Liouville's theorem). Fialkow has studied the conformal geometry of any curve not only in a euclidean space R_m but also in any riemann space V_m. See his paper, *Conformal geometry of curves*, Trans. Amer. Math. Soc. vol. 51 (1942).

[2] Kasner, *Conformality in connection with functions of two complex variables*, Trans. Amer. Math. Soc. vol. 48 (1940) pp. 50–62.

[3] See Kasner and DeCicco, *Pseudo-conformal geometry: Functions of two complex variables*, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 317–328.