

$\Delta_h^2 f(x) = f(x+h) + f(x-h) - 2f(x) = o(h)$ uniformly in x . If $\Delta_h^2 f = O(h)$, $f \in \Lambda^*$. If $f \in \text{Lip } 1$, then $f \in \Lambda^*$. The converse is false, since there are $f \in \Lambda^*$ nowhere differentiable. The modulus of continuity of an $f \in \Lambda^*$ is $O(\delta \log \delta)$. The class Λ^* is sometimes more natural than Lip 1. (i) A necessary and sufficient condition that the best approximation $E_n[f] = O(n^{-k})$ ($k=1, 2, \dots$) is $f^{(k-1)} \in \Lambda^*$. (ii) If $f \in \Lambda^*$, so does the conjugate function \tilde{f} . (iii) Let f^α, f_α denote the α th derivative and integral of f ($0 < \alpha < 1$). If $f \in \Lambda^*$, then $f^\alpha \in \text{Lip } (1-\alpha)$. If $f \in \text{Lip } \alpha$ then $f_{1-\alpha} \in \Lambda^*$. (iv) A necessary and sufficient condition that a harmonic function $f(r, x)$, $0 \leq r < 1$, be the Poisson integral of an $f \in \Lambda^*$ is $\partial^2 f(r, x) / \partial x^2 = O\{1/(1-r)\}$. If $\Delta_h^2 f(x) = o(h)$ for each x , f is smooth ($f \in \lambda$). A $g(x)$ defined over a set E is said to satisfy condition D , if $g(x)$ takes all intermediate values. (v) If $f \in \lambda$, $f'(x)$ exists in a dense set (Rajchman) and satisfies condition D . (vi) The sum of a trigonometric series with coefficients $o(1/n)$ satisfies condition D . (vii) If $g(x)$ is continuous, $\tilde{g}(x)$ satisfies condition D . (Received October 28, 1944.)

APPLIED MATHEMATICS

24. C. H. Dix, C. Y. Fu, Mrs. E. W. McLemore: *The cubic Rayleigh wave equation.*

Consider a plane compressional wave incident on the free plane surface of a semi-infinite elastic medium. The incident and reflected compressional amplitudes are respectively A and B . Then $B/A = N(i, s)/D(i, s)$ where $s = \lambda/\mu$, $ND = 16(s+1)w^8 - 8(3s+4)w^2 + 8(s+2)w - (s+2)$ and $w = \sin^2 r_1$, $r_1 =$ reflection angle of shear wave. Zeros of ND show two or no i 's (also zeros of N) corresponding to no reflected compressional wave. The third zero is a zero of D and gives the reciprocal of the important solution of the Rayleigh wave cubic. The cubic curves all pass through one fixed point whose coordinates are $w = 1.0957$, $ND = -1.83927$. For a fixed s , ND has the same value when $w = 0$ that it has when $w = 1$. If s corresponds to a small Poisson ratio then i for the larger zero of N will be very close to 90° , giving no reflection of compressional type whereas for $i = 90^\circ$ all reflected energy is compressional. There is a discontinuity when $s = 0$ where $B/A = +1$ while $B/A = -1$ for s greater than 0. (Received October 27, 1944.)

25. H. W. Eves: *A geometrical note on the isocenter.*

Consider a central projection of plane p on plane p' , L being the center of projection, and adopt the convention that angular directions on p (or p') are positive if they are counterclockwise when p (or p') is viewed from L . A point on p is called a positive isocenter on p if all angles on p having the point for vertex are invariant under the projection, and a point on p is called a negative isocenter if all angles on p having the point for vertex project into equal but oppositely directed angles on p' . It is shown that a tilted photograph possesses one and only one positive isocenter and one and only one negative isocenter. These points are geometrically located on the picture. Now the positive isocenter has long been known in photogrammetry, but the existence of the negative isocenter does not seem to have been noticed before. With the combined aid of these two isocenters a simple graphical procedure is developed for rectifying a tilted photograph. The mapping process can then be continued by the method of radial plotting. (Received October 19, 1944.)

26. H. W. Eves: *Analytical and graphical rectification of a tilted photograph.*

An aerial photograph fails to be a perfect map of the ground photographed be-