ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial num- of the abstract.

Algebra and Theory of Numbers

1. H. W. Becker: *The composite umbra theorem.*

Let $U, V, W, \cdots$ be different umbrae, and let $[\ ]$ confine a polynomial umbra. It is well known that $[U+V]_n=(U+V)^n$, based on the generator equation, $e^{[U+V]}=e^U\cdot e^V\cdot[UV]_n$, needs analogous definition. $e^{[UV]}=e^{UV}=(e^{eU-1})^{(V)}=e^{U\epsilon^*}(V)$, where $(V)_n$ is a Jordan factorial, and $[U\epsilon]_n=\phi_n(U)$ is the exponential polynomial of E. T. Bell (Ann. of Math. vol. 35 (1934) p. 263). Then $[UV]_n=[U\epsilon^*(V)]_n=\phi_n(U^*V)$, where * means that every term in U of weight m is multiplied by $(V)_m$. This is the composite umbra theorem. Such asymmetric composition is in general commutative, associative and distributive only for scalars, or for umbra iterates and inverses (calculated from $UU=U^{(2)}$, $UU^{(-1)}=1$, and so on). These decompositions greatly simplify, if $U_0=U_1=1$. Or they may be generalized, to an umbra form $[fU]_n$, where f is any function of any number of umbrae. Where the U are scalars, $[fU]_n$ reduces to $(fU)^n$, conveniently verifying the theorem and its consequences. The extension to any number of factors, $[UVW\cdots]$, is in close parallelism with the iterated exponential integers of E. T. Bell (Ann. of Math. vol. 39 (1938) p. 539), the classic instance. (Received October 28, 1944.)

2. H. W. Becker: *The hyper-umbra theorem.*

An umbra U is the representative of a series $U_0, \cdots, U_n, \cdots$. The umbra of an umbra, and so on, to m dimensions, or blanks, is called a hyper-umbra, and written ${}^mU={}^mU\{,\cdots,\}$. The fundamental umbra is ϵ, of generator $e^{e^t-1}=e^t\epsilon$. Its property $(\epsilon)=1$, where $(\ )$ is a Jordan factorial, underlies the new operational transformation $e^E=eE^\epsilon$ in the finite difference calculus. Application to an umbra yields $e^{E}{}^0U\{0\}=eU\{\epsilon\}$. The classic instance is Dombinski's theorem, in the form $e^E0^r=e\epsilon_r$. The operation may be iterated, along each dimension of a hyper-umbra. Denote by ${}^me$ a continued exponential of the mth order. Then ${}^me^{E}{}_0{}^mU\{0,\cdots,0\}={}^me^{mU}=e^{m}\cdot{}^mU\{\epsilon,\cdots,\epsilon\}$. This is the hyper-umbra theorem. Where ${}^mU=Y$ is the cubic array whose typical cell is $(\overline{UZ}+X)^n$, this gives ${}^3e^Y=\exp\exp\exp Y=e^e\cdot e^{e^U}$. Where ${}^mU=W$ is the square array of cells $\overline{UZ}_n=(U+\cdots+U)^n$ to ZU's, $e^{e^W}=e^{e^U}$. This is remarkable, in that the part is equipotent to the whole. If $U=1=$ the identity umbra, then $W=M=$ the table of all integer powers. Thus the power matrix is equipotent to unity. The theorem generalizes to ${}^me^{TE}{}_0{}^mU\{0,\cdots,0\}={}^me^{mU\cdot T}=e^{mT*}\{\epsilon T,\cdots,\epsilon T\}$, where * denotes scalar or subscript multiplication according as T is ordinary or umbral. (Received October 28, 1944.)